Applied Mathematics and Computation 311 (2017) 66–84

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Stationary distribution and extinction of the DS-I-A model disease with periodic parameter function and Markovian switching

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ARTICLE INFO

MSC:
60H10
34F05

Keywords:
DS-I-A models
Stochastic periodic solutions
Stationary distribution
Markov switching
Ergodicity

ABSTRACT

This paper introduces the DS-I-A model with periodic parameter function and Markovian switching. First, we will prove that the solution of the system is positive and global. Furthermore, we draw a conclusion that there exists nontrivial positive periodic solution for the stochastic system and we establish sufficient conditions for extinction of system. Moreover, we construct stochastic Lyapunov functions with regime switching to obtain the existence of ergodic stationary distribution of the solution to DS-I-A model perturbed by white and telephone noises and we also establish sufficient conditions for extinction of system with regime switching. Finally, we test our theory conclusion by simulations.

1. Introduction

The mathematical models played an important role in examining the characteristics of infectious diseases since the pioneer work of Kermack and McKendrick [1]. It provides us useful control measures from [2,3]. Famous models of infectious disease population dynamics [1,2,4] already exist in literature. A simple homogeneous AIDS model is given by the following system of ODEs [5]:

$$\begin{cases} \dfrac{dS_i(t)}{dt} = \mu(S_i^0 - S_i(t)) - \dfrac{\beta\alpha_i S_i(t)I(t)}{N(t)}, \qquad 1 \le i \le n, \\ \dfrac{dI(t)}{dt} = \sum_{i=1}^{n} \dfrac{\beta\alpha_i S_i(t)I(t)}{N(t)} - (\mu+\gamma)I(t), \\ \dfrac{dA(t)}{dt} = \gamma I(t) - \delta A(t), \end{cases} \tag{1.1}$$

where $N(t) = \sum_{k=1}^{n} S_k(t) + I(t), S_i(t) (i = 1, 2, \dots, n)$ denotes the n individuals susceptible to infection subgroups; $I(t)$ the infected individuals; $A(t)$ the AIDS cases; $\mu S_i^0 (i = 1, 2, \dots, n)$ the input flow into the n susceptible subgroups; $\alpha_i (i = 1, 2, \dots, n)$ the susceptibility of susceptible individuals in subgroup i and $\frac{\beta I(t) S_i(t)}{N(t)}\alpha_i$ the standard incidence ratio of susceptible sub-

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http://dx.doi.org/10.1016/j.amc.2017.04.029
0096-3003/

groups S_i; μ the natural mortality rate; γ the removal rate coefficient of the infected individuals and δ the sum of natural mortality rate and mortality due to illness.

Since the dynamics of group A has no effect on the disease transmission dynamics, thus we only consider

$$\begin{cases} \dfrac{dS_i(t)}{dt} = \mu(S_i^0 - S_i(t)) - \dfrac{\beta\alpha_i S_i(t)I(t)}{N(t)}, \qquad 1 \le i \le n, \\ \dfrac{dI(t)}{dt} = \displaystyle\sum_{i=1}^{n} \dfrac{\beta\alpha_i S_i(t)I(t)}{N(t)} - (\mu+\gamma)I(t). \end{cases} \tag{1.2}$$

The threshold conditions can be calculated which determine whether an infectious disease will spread in susceptible population when the disease is introduced into the crowed, according to research the disease free equilibrium $E_0(S_1^0, S_2^0, \dots, S_n^0, 0)$ of system (1.2) in [6].

And they obtain

$$R_0 = \frac{\beta\sum_{i=1}^{n}\alpha_i S_i^0}{(\mu+\gamma)\sum_{i=1}^{n} S_i^0},$$

where $R_0 < 1$, E_0 is local asymptotic stabile and disease extinct. When $R_0 > 1$ then E_0 is unstable and the disease will persistent existence (see [5]). The effective contact rate of infected individual in subgroup $S_i (i = 1, 2, \dots, n)$ is $\alpha_i\beta (i = 1, 2, \dots, n)$. Thus for initial time $(S_i = S_i^0)$, the average effective contact rate of infected individual in subgroup $S_i (i = 1, 2, \dots, n)$ is $\frac{\beta\sum_{i=1}^{n}\alpha_i S_i^0}{\sum_{i=1}^{n} S_i^0}$. $\frac{1}{\mu+\gamma}$ the average disease period of infected individuals. So R_0 is basic reproductive number.

It is well recognized fact that real life is full of randomness and stochasticity. Hence, the epidemic models are always affected by the environmental noise (in cite [7–14]). In [15–23], the stochastic models may be more convenient epidemic models in many situations.

There are different approaches to introduce random perturbations in the model both from biological and mathematical perspectives [24,25]. Then corresponding to system (1.2), one has the following stochastic model

$$\begin{cases} dS_i(t) = \left[\mu(S_i^0 - S_i(t)) - \dfrac{\beta\alpha_i S_i(t)I(t)}{N(t)}\right]dt + \sigma_i S_i(t)dB_i(t), \qquad 1 \le i \le n, \\ dI(t) = \left[\displaystyle\sum_{i=1}^{n} \dfrac{\beta\alpha_i S_i(t)I(t)}{N(t)} - (\mu+\gamma)I(t)\right]dt + \sigma_{n+1}I(t)dB_{n+1}(t), \end{cases} \tag{1.3}$$

where $B_i(t) (i = 1, 2, \dots, n)$ are independent standard Brownian motions with $B_i(0) = 0 (i = 1, 2, \dots, n)$ and $\sigma_i^2 > 0 (i = 1, 2, \dots, n)$ denote the intensities of the white noise. Other parameters are the same as in system (1.2).

On the other hand, many infectious of humans fluctuate over time and often show seasonal patterns of incidence. Taking account of periodic variation in epidemic models and studying the existence of periodic solutions are important and interesting to predict and control the spread of infectious diseases. Many results on the periodic solution of epidemic models have been reported [26–28] by using Has'minskii theory of periodic solutions and constructing suitable Lyapunov functions.

Motivated by above facts, in this paper, we will first consider the following stochastic DS-I-A model:

$$\begin{cases} dS_i(t) = \left[\mu(t)(S_i^0(t) - S_i(t)) - \dfrac{\beta(t)\alpha_i(t)S_i(t)I(t)}{N(t)}\right]dt + \sigma_i(t)S_i(t)dB_i(t), \qquad 1 \le i \le n, \\ dI(t) = \left[\displaystyle\sum_{i=1}^{n} \dfrac{\beta(t)\alpha_i(t)S_i(t)I(t)}{N(t)} - (\mu(t)+\gamma(t))I(t)\right]dt + \sigma_{n+1}(t)I(t)dB_{n+1}(t), \end{cases} \tag{1.4}$$

in which the parameter functions $\mu, S_i^0, \sigma_i, \beta, \alpha_i, \gamma, i = 1, 2, \dots, n,$ are positive, non-constant and continuous functions of period T.

Besides white noise, epidemic models may be disturbed by telephone noise which makes population systems switch from one regime to another. Let us now take a further step by considering another type of environmental noise, namely, color noise, say telegraph noise (see Refs. [29,30]). The telegraph noise can be illustrated as a switching between two or more regimes of environment, which differ by factors such as nutrition or as rain falls [31–34]. The switching is memoryless and the waiting time for the next switch has an exponential distribution. Therefore, we also consider the following stochastic