Applied Mathematics and Computation 311 (2017) 85–99

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

An implicit three-dimensional fractional step method for the simulation of the corona phenomenon

Andrea Villa[a,*], Luca Barbieri[a], Marco Gondola[a], Andres R. Leon-Garzon[b], Roberto Malgesini[a]

[a] Ricerca Sul Sistema Energetico (RSE), Via Rubattino 50, 20134 Milano, Italy
[b] CMIC Department "Giulio Natta", Politecnico di Milano, Piazza Leonardo da Vinci 32, 20133 Milano, Italy

ARTICLE INFO

2010 MSC:
35L65
76X05

Keywords:
Simulation
Corona
Streamer

ABSTRACT

The modeling of the corona effect has many technological applications especially in the power industry. The reduction of the computational burden of three dimensional simulations is a key factor in this area. Stability requirements may impose unacceptable constraints in three dimensions leading to huge computational costs. In this paper we develop an effective time-splitting method that remains stable and positive even when relatively large time steps and coarse meshes are used. We analyze the theoretical properties of the method and validate our approach against already published experimental data.

1. Introduction

In this paper we develop an effective three dimensional implicit scheme for the solution of the corona phenomenon [1]. The simulation of the inception of electric discharges is a very important topic with applications to many technological fields such as the treatment of surfaces [2,3], the diminution of air pollutants [4], the production of chemically active species [5] and the analysis of partial discharges [6].

The three-dimensional simulation of the corona effect and of the streamers is usually linked to huge computational costs, therefore this challenge has been tackled in a few cases such as [7,8]. In many other works some major simplifications have been adopted, for instance, one of the crudest approximations are the 1.5d models. These have been employed in [9] to simulate the corona effect and in [10] to estimate the streamer propagation. Another very popular approximation is the two dimensional axis-symmetric one. It has been used to predict the first phases of the streamer propagation [11,12] and to model the Trichel phenomenon on needles [13–15]. There are also a few cases of planar two dimensional streamer simulations [16,17] mainly devoted to the study of the instabilities of ionization fronts.

The computational burden associated to a full three dimensional modeling is further exacerbated by the fact that, in many cases, the stability requirements may request a strict mesh spacing and small time steps. In [18] it has been demonstrated that the coupling between the electrostatic equation and the movement of charges may cause some numerical instabilities even when relatively small time steps are used. The methods that implement a stable coupling between these two physical models are called asymptotic preserving methods, see [19–21].

* Corresponding author.
E-mail addresses: andrea.villa81@fastwebnet.it, andrea.villa@rse-web.it (A. Villa), luca.barbieri@rse-web.it (L. Barbieri), marco.gondola@rse-web.it (M. Gondola), andresricardo.leon@polimi.it (A.R. Leon-Garzon), roberto.malgesini@rse-web.it (R. Malgesini).

http://dx.doi.org/10.1016/j.amc.2017.04.037
0096-3003/

Moreover in [22] it has been shown that a large class of finite volume methods may be unstable when they are used to simulate the avalanche phenomenon. The electron avalanche constitutes an important part of the streamer-corona model. If no stabilization technique is used, the stability of that class of methods is linked to the mesh spacing. The methods that are stable whatever mesh spacing is used are called, through this paper, avalanche-stable.

Some other stability issues may include the Courant Friedrichs Lewy (CFL) condition [23] or the instability related to convection dominated drift-diffusion problems [24].

In this work we aim to develop a method whose numerical burden is not strictly constrained by stability issues. A huge number of numerical methods have been proposed to solve the corona or the streamer model, many of them are based on explicit finite volume techniques [25,26] and, therefore, they are constrained by the CFL condition. Also some higher order Discontinuous Galerkin methods [27] show very similar constraints. The commercial codes, like the one used in [13], usually implement full time-implicit techniques. As it has been shown in a number of works [18–20] implicit and semi-implicit time stepping schemes are usually asymptotic preserving. However, commercial codes are in almost all cases not designed to be avalanche-stable and must implement a strong mesh refinement to gain stability. Among the avalanche stable methods we mention the characteristics method [28] and the particle in a cell (PIC) method [8]. Unfortunately, to the best of the authors' knowledge, no asymptotic preserving PIC methods have been proposed so far.

Here we propose an implicit evolution of the explicit finite volume asymptotic preserving techniques proposed in [18] embedding the stabilization techniques developed in [22] for the avalanche phenomena. The implicit scheme we are going to introduce is no longer subjected to the CFL. This is a critical feature for the simulation of the Trichel corona effect since the time step should be adapted to cope with the fast variations of the discharges and the almost flat variations of the inter-discharge periods. The satisfaction of the CFL condition would impose unacceptable time step constraints between one discharge and the other. The stability requirements we have listed are crucial when long simulation runs are performed. To the best of our knowledge, no numerical method, currently available, embeds all these characteristics. In this work we develop a method that has the properties we are looking for and we analyze it: in particular we concentrate on the stability issues and on the positivity of the method.

Let us now review the structure of this paper. In Section 2.1 we introduce the corona model and its discretization. The numerical properties of the method are analyzed in Section 3 and in Section 4 we discuss the implementation of the algorithm. Finally, in Section 6, we compare the results with some data found in the literature.

2. The model and its discretization

2.1. The model

Let $\Omega \subset \mathbb{R}^3$ be an open domain with Lipschitz boundary $\partial\Omega$, let $\vec{n}$ be the outward pointing unit vector on the boundary and let $[0, T]$ be a time interval where T is the final time. The spatial coordinate vector is denoted with $\vec{x} \in \Omega$ and the time variable with $t \in [0, T]$. We consider the standard corona problem [13,29]:

$$\begin{cases} \frac{\partial n_e}{\partial t} - \vec{\nabla}\cdot\left(n_e\mu_e\vec{E}\right) - \vec{\nabla}\cdot\left(d_e\vec{\nabla}n_e\right) = (\alpha-\eta)\mu_e\left|\vec{E}\right|n_e - \beta n_p n_e + S_{ph}, \\ \frac{\partial n_p}{\partial t} + \vec{\nabla}\cdot\left(n_p\mu_p\vec{E}\right) = \alpha\mu_e\left|\vec{E}\right|n_e - \beta n_p n_e - \beta n_p n_n + S_{ph}, \\ \frac{\partial n_n}{\partial t} - \vec{\nabla}\cdot\left(n_n\mu_n\vec{E}\right) = \eta\mu_e\left|\vec{E}\right|n_e - \beta n_p n_n, \\ \vec{\nabla}\cdot\vec{E} = \frac{e}{\varepsilon_0}(n_p - n_n - n_e), \\ \vec{E} = -\vec{\nabla}\phi, \end{cases} \tag{1}$$

where $n_e(t,\vec{x})$ is the concentration of the electrons, $n_p(t,\vec{x})$ is the concentration of the positive ions, $n_n(t,\vec{x})$ is the concentration of the negative ions, $\vec{E}(t,\vec{x})$ is the electric field, $\phi(t,\vec{x})$ is the electric potential, with $\vec{x}\in\Omega$ and $t \in [0, T]$. e is the electron charge, ε_0 is the vacuum permittivity, μ_e, μ_p, μ_n, are, respectively, the mobilities of the electrons, positive and negative ions, $\alpha(|\vec{E}|)$, $\eta(|\vec{E}|)$, β are, respectively, the ionization, attachment, and recombination coefficients and d_e is the diffusion coefficient. The mobilities and the reaction coefficients can be provided by the swarm parameter databases such as in [1,30,31]. We point out that the mobilities and the reaction coefficients are all positive.

System (1) must be complemented by a set of initial and boundary conditions. For these latter therefore we define:

$$\begin{aligned} \partial\Omega_I^P(t) &= \left\{\vec{x}\in\partial\Omega : \vec{E}(t,\vec{x})\cdot\vec{n}(\vec{x}) < 0\right\}, \quad \partial\Omega_O^P(t) = \left\{\vec{x}\in\partial\Omega : \vec{E}(t,\vec{x})\cdot\vec{n}(\vec{x}) \geq 0\right\}, \\ \partial\Omega_I^N(t) &= \left\{\vec{x}\in\partial\Omega : \vec{E}(t,\vec{x})\cdot\vec{n}(\vec{x}) > 0\right\}, \quad \partial\Omega_O^N(t) = \left\{\vec{x}\in\partial\Omega : \vec{E}(t,\vec{x})\cdot\vec{n}(\vec{x}) \leq 0\right\}, \end{aligned} \tag{2}$$

and we set the following boundary conditions:

$$n_e(t,\vec{x}) = n_{e,b}(t,\vec{x}),\ \vec{x}\in\partial\Omega, \quad n_n(t,\vec{x}) = n_{n,b}(t,\vec{x}),\ \vec{x}\in\partial\Omega_I^N(t), \quad n_p(t,\vec{x}) = n_{p,b}(t,\vec{x}),\ \vec{x}\in\partial\Omega_I^P(t), \tag{3}$$

where $n_{e,b}$, $n_{p,b}$ and $n_{n,b}$ are, respectively, the positive boundary electron, positive and negative ion concentration functions. We include the secondary electron emission phenomenon and we compute the boundary electron concentration as $n_{e,b}(t,\vec{x}) = \gamma\frac{\mu_p(t,\vec{x})}{\mu_e(t,\vec{x})}n_{p,b}(t,\vec{x})$ where γ is the secondary emission coefficient. The boundary conditions associated to the