Applied Mathematics and Computation 311 (2017) 100–117

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Existence, uniqueness, and exponential stability analysis for complex-valued memristor-based BAM neural networks with time delays☆

Runan Guo[a], Ziye Zhang[a,*], Xiaoping Liu[b], Chong Lin[c]

[a] *College of Mathematics and Systems Science, Shandong University of Science and Technology, Qingdao 266590, China*
[b] *School of Information and Electrical Engineering, Shandong Jianzhu University, Jinan 250101, China*
[c] *Institute of Complexity Science, Qingdao University, Qingdao 266071, China*

A R T I C L E I N F O

Keywords:
Exponential stability
Memristor-based BAM neural networks
Complex-valued systems
Time delays
Lyapunov functional
M-matrix

A B S T R A C T

This article explores the exponential stability problem of complex-valued bidirectional associative memory (BAM) neural networks with time delays. This analysis is on the basis of the *M*-matrix approach, the differential inclusions theory and the homeomorphism property. By constructing a novel Lyapunov functional, a sufficient criterion for the existence, uniqueness, and exponential stability for the equilibrium point of the considered system is derived. Moreover, similar results in terms of *M*-matrix are also obtained for the exponential stability problem of delayed complex-valued BAM neural networks without memristors. In the end, two numerical examples are provided to demonstrate the availability of the obtained results.

1. Introduction

The bidirectional associative memory (BAM) neural networks model originally presented in [1] has special structure of connection weights. Since it is regarded as an extension of the unidirectional autoassociator of Hopfield neural networks, it shows more excellent characteristics and wide application in science and engineering fields. As a result, more and more researchers are concerned with studying BAM neural networks and lots of achievements for various dynamical behaviors of BAM neural networks have been developed [2–7]. For instance, the stability problem of BAM neural networks with constant leakage delays is discussed by applying the properties of *M*-matrices in [2]. Based on sampled-data control, the stabilization problem of BAM neural networks with time-varying delays in the leakage terms is analyzed in [3]. In [7], LMI-based sufficient criterion is established to assure the exponential stability for delayed neutral BAM neural networks via new inequalities methods.

On the other hand, in electronic circuit theory, the fourth circuit element named memristor that was originally developed by Chua [8] is a two-terminal nonlinear device, which is different from resistor, capacitor, inductor. In nonvolatile memory storage, the memristor becomes a very hot topic for its potential applications after the build of a practical memristor model

☆ This work was supported in part by the National Natural Science Foundation of China (61503222, 61673227), in part by NSERC, Canada, in part by the Research Fund for the Taishan Scholar Project of Shandong Province of China, in part by the Project funded by China Postdoctoral Science Foundation (2016M602166), and in part by the Fund for Postdoctoral Applied Research Projects of Qingdao (2016116).

* Corresponding author at: College of Mathematics and Systems Science, Shandong University of Science and Technology, Qingdao 266590, China.
E-mail address: zhangzy02@126.com (Z. Zhang).

http://dx.doi.org/10.1016/j.amc.2017.05.021
0096-3003/

[9,10]. Moreover, it has many characteristics, just like as the neurals in the human brain. Thus, one can emulate the huaman brain by using the new circuit devices and structuring the memristor-based neural networks. But, since memristive neural networks reveals nonlinear state-dependent switching behaviors, it becomes more difficult and more challenging to analyze it, which attracts many scholars' attentions and plenty of results have appeared, see [11–20] and references therein. Recently, new memristor-based BAM neural networks are built by replacing the connection weights in BAM neural networks with those with memristors concepts. Then, a series of results for this model have been reported [21–25]. In [22] and [23], the passivity problems of delayed memristive BAM neural networks and delayed stochastic memristive neutral-type BAM neural networks are discussed, respectively. Synchronization and finite-time Mittag-Leffler synchronization of memristive BAM delayed neural networks with fractional-order derivatives are studied in [24] and [25], respectively.

The research for complex-valued neural networks with complex-valued states, connection weights and activation functions, has been always a rapidly growing issue due to their more complicated properties and practical applications on engineering areas, such as optoelectronics, speech synthesis, and pattern recognition [26–31]. Further more, comparing with real-valued neural networks, complex-valued systems can deal with many difficulties that real-valued ones cannot do. Such as, the XOR problem solved by one complex-valued neuron [32] and the speed and direction as an entire complex vector in wind profile model [33]. Therefore, it is very significant to study complex-valued neural networks, especially, the dynamical behaviors of this model. Hitherto, numerous related articles have been published [34–48]. Moreover, for complex-valued BAM neural networks and complex-valued memristor-based neural networks, there have also been many achievements about dynamical behaviors of these systems [49–58]. For instance, in [49] and [50], the global asymptotic stability problems for delayed complex-valued BAM neural networks and complex-valued Cohen-Grossberg delayed BAM neural networks are studied, respectively. Sufficient criteria of the passivity for complex-valued memristor-based neural networks with time-varying delays are presented in [53,54]. The global exponential dissipativity for complex-valued memristive delayed neural networks are analyzed in [55,56]. However, to the best of our knowledge, there is still no information in the published literature about the dynamical behavior analysis for delayed complex-valued nemristor-based BAM neural networks. This paper aims to do this work.

Inspired by the above analysis, in this paper, the problem of the existence, uniqueness, and exponential stability of the equilibrium point for memristor-based complex-valued BAM neural networks with time delays is investigated. The main contributions of our work involve the following points.

(1) It is the first time that the exponential stability of memristor-based complex-valued BAM neural networks with time delays is studied.
(2) Based on the differential inclusions theory, the homeomorphism theory and M-matrix method, by constructing a novel Lyapunov functional, a sufficient condition to assure the existence, uniqueness, and exponential stability of the equilibrium point for the considered system is derived. Moreover, in contrast to the existing results, the proposed function is different and improved.
(3) When the considered system reduces delayed complex-valued BAM neural networks without memristors, the similar results can be also obtained.
(4) For complex-valued activation functions, more general assumptions are provided in this paper. So our work can serve a broader class of systems.

The rest part is summarized as follows. In Section 2, the system description and some preliminaries are involved. The main results are presented in Section 3. In Section 4, numerical simulations are provided. Section 5 gives the conclusion.

2. Problem formulation and preliminaries

Consider the complex-valued memristor-based BAM neural networks described as the following form:

$$\begin{cases}\dot{u}_p(t) = -d_{1p}u_p(t) + \sum_{q=1}^{m} a_{1qp}(u_p(t))f_q(v_q(t)) + \sum_{q=1}^{m} b_{1qp}(u_p(t))f_q(v_q(t-\tau_{2q})) + I_{1p},\\ \dot{v}_q(t) = -d_{2q}v_q(t) + \sum_{p=1}^{n} a_{2pq}(v_q(t))g_p(u_p(t)) + \sum_{p=1}^{n} b_{2pq}(v_q(t))g_p(u_p(t-\tau_{1p})) + I_{2q},\end{cases} \tag{1}$$

where $p = 1, 2, \ldots, n$ and $q = 1, 2, \ldots, m$. $u_p(t)$ and $v_q(t)$ are the neuron state variables. I_{1p} and I_{2q} denote the pth and qth components of external inputs. $d_{1p} > 0$ and $d_{2q} > 0$ are constants. τ_{1p} and τ_{2q} are constant time delays. $f_q(v_q(t))$, $g_p(u_p(t))$, $f_q(v_q(t-\tau_{2q}))$, and $g_p(u_p(t-\tau_{1p}))$ denote the complex-valued activation functions. $a_{1qp}(u_p(t))$, $a_{2pq}(v_q(t))$, $b_{1qp}(u_p(t))$, and $b_{2pq}(v_q(t))$ are complex-valued connection memristive weights. They are defined as

$$\begin{aligned} a_{1qp}(u_p(t)) &= \frac{\mathcal{W}_{1qp}}{\mathcal{C}_{1p}} \times \mathrm{sgn}_{qp}, \quad b_{1qp}(u_p(t)) = \frac{\mathcal{M}_{1qp}}{\mathcal{C}_{1p}} \times \mathrm{sgn}_{qp},\\ a_{2pq}(v_q(t)) &= \frac{\mathcal{W}_{2pq}}{\mathcal{C}_{2q}} \times \mathrm{sgn}_{pq}, \quad b_{2pq}(v_q(t)) = \frac{\mathcal{M}_{2pq}}{\mathcal{C}_{2q}} \times \mathrm{sgn}_{pq},\end{aligned} \tag{2}$$

where $\mathrm{sgn}_{qp} = \mathrm{sgn}_{pq} = 1$ for $p \neq q$ and $\mathrm{sgn}_{qp} = \mathrm{sgn}_{pq} = -1$ for $p = q$. $\mathcal{C}_{1p}$ and $\mathcal{C}_{2q}$ are the capacitors. $\mathcal{W}_{1qp}$, $\mathcal{M}_{1qp}$, $\mathcal{W}_{2pq}$ and $\mathcal{M}_{2pq}$ are the memductances of memristors $\mathcal{R}_{1qp}$, $\mathcal{F}_{1qp}$, $\mathcal{R}_{2pq}$ and $\mathcal{F}_{2pq}$, respectively. $\mathcal{R}_{1qp}$ denotes the memristors between $f_q(v_q(t))$ and $u_p(t)$; $\mathcal{F}_{1qp}$ denotes the memristors between $f_q(v_q(t-\tau_{2q}))$ and $u_p(t)$; $\mathcal{R}_{2pq}$ denotes the memristors between