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Stability analysis of linear systems with interval time-varying delays utilizing multiple integral inequalities

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ABSTRACT

This paper is devoted to stability analysis of continuous-time delay systems with interval time-varying delays having known bounds on the delay derivatives. A parameterized family of Lyapunov–Krasovskii functionals involving multiple integral terms is introduced, and novel multiple integral inequalities are utilized to derive sufficient stability condition for systems with time-varying delays. The efficiency of the proposed method is illustrated by numerical examples.

1. Introduction

It is well-known that time-delays are frequently present in various physical, industrial and engineering systems. The delays may cause poor performance or even instability of systems, therefore much attention has been devoted to obtain tractable stability criteria for systems with time delay during the past few decades (see e.g., the monographs [1,2], some recent papers [3–27] and the references therein).

Let us consider the following system with time-varying delay:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + A_d x(t-\tau(t)), \\ x(t) &= \varphi(t), \qquad\qquad t \in [-\overline{\tau}, 0], \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^{n_x}$ is the state, and $A,\ A_d \in \mathbb{R}^{n_x \times n_x}$ are given matrices, the time-varying delay τ is supposed to be a differentiable function satisfying conditions

$$0 \le \underline{\tau} \le \tau(t) \le \overline{\tau}, \quad \underline{\mu} \le \dot{\tau}(t) \le \overline{\mu}, \quad \underline{\tau} < \overline{\tau} \tag{2}$$

with known bounds $\underline{\tau}$, $\overline{\tau}$, $\underline{\mu}$, $\overline{\mu}$, and function φ gives the initial condition.

Several approaches have been elaborated and successfully applied for the stability analysis of time delay systems (see the references above for excellent overviews). A major issue in this respect is to develop delay-dependent stability conditions, which provide as wide delay bounding interval guaranteeing stability as possible. Many important results on asymptotic stability of time-delay systems have been established using the Lyapunov–Krasovskii functional (LKF) approach.

In order to enhance the effectiveness, more and more involved Lyapunov–Krasovskii functionals have been introduced during the past decades involving single/multiple integral terms with respect to quadratic functions. On the other hand, much effort has been devoted to derive more and more tight inequalities (Jensen's inequality and different forms of

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Wirtinger's inequality [1,2,4,5,22,28], etc.) for the estimation of single, double and multiple quadratic integral terms in the derivative of the LKF. Lately, the so-called Bessel–Legendre inequality has been developed in [12] to deal with single integral terms of quadratic functions, and a wide range of multiple integral inequalities has been derived in [9–11]. Very recently, general integral inequalities that encompass all the above-mentioned inequalities are parallel presented in [16,17] based on orthogonal polynomials in different Euclidean spaces with integral inner products.

Utilizing these results, sufficient stability conditions for linear time delay systems partly with constant, partly with time varying delays are derived in the form of linear matrix inequalities (LMIs). We note that a great part of recent works investigates time-delay systems with bounded delays (i.e., when $\underline{\tau}=0$) and with different conditions on the delay derivative.

The aim of the present work is to investigate the stability problem of linear systems with interval time-varying delays (i.e., when $\underline{\tau}>0$) having known bounds of the delay derivative via the application of the inequalities proposed in [17].

It has to be emphasized that the time-variance of the delay causes some new problems compared to the case of constant delay: (1) new terms and different augmented variables have to be considered in the LKF for efficiency; (2) the presence of time-varying delays implies the non-convexity of the obtained matrix inequalities with respect to the length of the intervals, which necessitates special handling.

The contribution of the paper can be summarized as follows:

1. A parameterized family of LKFs is introduced involving multiple integral terms and augmented state variables that are chosen in accordance with the applied estimations. The possible choice of the parameters gives great flexibility to find a compromise between the computational complexity and the reduction of conservatism.
2. Relaxed sufficient LMI stability conditions for systems with interval time-varying and bounded time-varying delays are derived.
3. The derivation is based on the multiple integral inequalities published in [17].
4. It will be shown that the proposed condition yields less conservative results with substantially less number of decision variables than several others known from the literature.

The paper is organized as follows. Some preliminary results will be recalled in Section 2. The main result will be derived in Section 3, which starts by introducing a new Lyapunov–Krasovskii functional in Section 3.1. Next, the analysis problem will be investigated in Section 3.2. In Section 4, some numerical examples illustrate the effectiveness of the results. Finally, the conclusion will be drawn.

Standard notations are used. As usual, $P>0(\geq 0)$ denotes the positive (semi-)definiteness of P. Symbols $\mathbb{S}^n$ ($\mathbb{S}^n_+$) denote the set of symmetric (and positive definite) matrices of size $n\times n$. For any $A\in\mathbb{R}^{n\times n}$, symbol He($A$) is defined as He$(A)=A+A^T$. For the sake of brevity, asterisks replace the blocks in hypermatrices that are inferred readily by symmetry. If $n=0$ and/or $m=0$, then vector $x\in\mathbb{R}^n$ and matrix $C\in\mathbb{R}^{n\times m}$ are considered to be empty. The Euclidean vector norm in $\mathbb{R}^n$ is $\|\cdot\|$. We also denote $x_t(\theta)=x(t+\theta)$, $\dot{x}_t(\theta)=\dot{x}(t+\theta)$, $(\theta\in[-\overline{\tau},0])$.

2. Preliminaries

This paper aims to derive sufficient stability conditions in LMI form based on some multiple integral inequalities recently published in [17]. For convenience of the readers, we evoke these estimations in the following lemmas.

Lemma 1 *[17].* *Let $W\in\mathbb{S}^{\overline{n}}_+$, let $\mathbf{E}$ be a Euclidean space with the scalar product $\langle .,. \rangle$, and let $\pi_i\in\mathbf{E}$, $(i=0,1,\dots)$ form an orthogonal system. If $\nu\geq 0$ is a given integer, then for any $f\in\mathbf{E}^{\overline{n}}$, the following inequality holds*

$$J_W(f)=\langle f,Wf\rangle\geq\sum_{j=0}^{\nu}\frac{1}{\|\pi_j\|^2}w_j^TWw_j,$$

where $w_j=\langle f,\pi_j\rangle$, and the scalar product is taken componentwise.

Lemma 1 can be specified for the following case. Let $\mathbf{E}=L_2[a,b]$ with a scalar product defined by

$$\langle g_1,g_2\rangle_{\ell,[a,b]}=\int_a^b\left(\frac{s-a}{b-a}\right)^{\ell}g_1(s)g_2(s)ds,\qquad g_1,g_2\in L_2[a,b],\tag{3}$$

where $\ell\geq 0$ is a given integer. It is well-known that this scalar product can also be written as

$$\langle g_1,g_2\rangle_{\ell,[a,b]}=\frac{\ell!}{(b-a)^{\ell}}\int_a^b\int_{v_1}^b\dots\int_{v_\ell}^b g_1(s)g_2(s)ds dv_\ell\dots dv_1.$$

We shall consider the set of polynomials $p_{\ell,\,n}$ of degree $n=0,1,2,\dots,$ orthogonal with respect to the scalar product (3) satisfying relations

$$\|p_{\ell,n}\|^2_{\ell,[a,b]}=\frac{b-a}{\ell+2n+1},\qquad p_{\ell,n}(a)=(-1)^n\frac{\ell+n}{n},\qquad p_{\ell,n}(b)=1.$$

(Further properties are given in [17].) In particular, we shall use the notation $P_{\ell,\,n}$, if $[a,b]=[0,1]$. Making calculations, it may be useful to express the polynomials $P_{\ell,\,n}$, for $n=0,1,\dots,K$ in terms of $\{1,\ x,\ ,\dots,x^K\}$. To this end, matrix $G(\ell,K)\in$