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Inverse engineering preferences in simple games

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ABSTRACT

A method for inverse engineering decision-makers' preferences based on observable behaviour is designed. This technique allows analysts to narrow down the list of potential preference rankings of possible states in a conflict for each decision-maker using probabilities and expected values. During the inverse engineering procedure, the list of all possible preference rankings is narrowed as decision-makers move and counter-move. Accurate preference information is key to building quality conflict and game models; however, preference rankings for decision-makers are often difficult to obtain directly. A simple two decision-maker, four-state game is used to demonstrate the applicability of the method and to illustrate the insights it provides.

1. Introduction

Strategic interactions among decision-makers (DMs) are common to a wide variety of contexts, including economics, evolutionary biology, operations research, negotiations, and military science. Game theory is the study of such interactions; it provides a rich array of mathematical tools to model, predict, and analyse conflicts. Since its introduction by von Neumann and Morgenstern [1], classical game theory has seen many of its original assumptions re-examined and re-evaluated. The notion of perfect rationality has ceded ground to that of bounded rationality [2,3]; a "static" approach has given way to a dynamic, time dependent approach [4]. Modern approaches are based on evolutionary game theory which combines bounded rationality with dynamics to analyse conflicts and their evolution [5,6]. Recent developments in evolutionary game theory include multigames, in which each DM in a population has different payoffs [7,8], as well as graph and network approaches in which social networks and spatial games are often represented using graph or lattice structures with edge weights representing utilities [9–12].

Traditionally, DM preferences are modelled with utility functions. Such functions are meant to capture the benefit (utility) that each DM derives from a particular state or outcome; many approaches seek to maximise the utility functions for the DM(s) in question. Cardinal utilities and preferences can, however, be replaced by ordinal preferences in which states are pairwise ranked and compared.

The Graph Model for Conflict Resolution (Graph Model, for short) is a well-known methodology for analysing complex conflicts involving several DMs which uses ordinal preferences [13,14]. Using DMs, possible states, and the preferences of each DM over the set of states, the Graph Model provides DMs and analysts with informative strategic insights into the conflict under study. For a conflict with a set N of DMs and a set S of possible states, the Graph Model depicts the conflict as a finite set of directed graphs $D_i = (S, A_i)$ with $i \in N$. The vertices of the graph represent the set of states and an arc

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A_i between two states indicates that DM i can unilaterally move between them by changing the selection of options under their control. Several solution concepts are defined in the Graph Model to reflect interactive human behaviour under conflict; the most commonly used are Nash [15,16], general metarational [17], symmetric metarational [17], and sequential stabilities [18].

Of the "ingredients" required by the Graph Model, preferences are arguably the most difficult to obtain. The discovery of each DM's preferences is important for analysing both historical and ongoing conflicts as the analytical results depend on their accuracy. For ongoing conflicts, a DM may not be willing to share his or her preferences with an analyst and will be much less willing to do so with an opponent; the study of historical conflicts, on the other hand, may be hampered by a lack of information or reliable sources. There is also the possibility that a DM may be uncertain about their own preferences, taking actions but perhaps not clearly understanding why.

Accurate preference information is crucial to DMs and analysts who wish to garner useful and informative insights about a conflict. Although preferences can be difficult to ascertain, opponent moves are clear and observable. Since moves are informed by a DM's preferences, the transparency of an opponent's moves can be leveraged by a DM to infer that opponent's preferences. In this work, a method for ascertaining DM preferences using observed behaviour is proposed. The method narrows down the set of a DM's possible preference rankings using the information provided by that DM's actions. The results obtained from this technique could be used to produce more accurate insights when using the Graph Model.

This paper is organized as follows: Section 2 provides an overview of related work; Section 3 outlines the problem and sets up the solution; Section 4 analyses a simple 2-DM, 2-option game using the proposed methodology; Section 5 provides conclusions and future research avenues.

2. Related work

Preferences in the Graph Model have been studied from many perspectives including unknown preferences [19,20], fuzzy preferences [21], probabilistic preferences [22], grey-based preferences [23], strength of preferences [24–26], and information-gap models [27]. The first four of these approaches examine different preference relations and structures in order to deal with the uncertainty that a particular DM has over the set of states. Strength of preferences considers "strongly preferred" relations between states in addition to the usual "preferred" and "indifferent" relations [24,25]; this concept was later generalized to any level of preference [26] and incorporated into coalition analysis [28].

Unknown preferences make use of an additional "unknown" relation between possible states. Unknown preferences and strength of preference are combined in [20] to produce a hybrid preference structure. Fuzzy preferences make use of fuzzy sets, which specify a degree of belonging of an element to a set, to define fuzzy relations and fuzzy preferences [21]. Probabilistic preferences assume that DMs have probabilistic preferences over the set of states, whereby one state is preferred over another with a certain probability [22]. Grey-based preferences use grey numbers, which are real numbers that may be a member of a discrete set of real numbers, or may fall within one or several intervals [23]. In all cases, the new preferences structures are used to construct their respective analogues of Graph Model solution concepts and equilibria.

The common assumption underlying most of these methods is that the DMs are uncertain of their or their opponents' ranking of states; some states may be strictly preferred over others, but some ambiguity may exist. In other words, the uncertainty may be present within each DM's own preference ranking. This research examines preferences from a different perspective: rather than accounting for uncertainty within each DM's preferences, DMs are uncertain about their own preferences, their opponent's preferences, or both. The fundamental question is whether preferences can be inferred based on behaviour. Rather than modelling preferences using different preference structures (unknown, fuzzy sets, probabilistic, grey numbers, or strength of preference), a simple underlying structure is assumed and a DM's observable actions are used to narrow the list of feasible rankings. It should be noted that the Graph Model is able to handle intransitive and pairwise preferences; these considerations fall beyond the scope of this paper, which assumes that preferences are transitive.

Incomplete information in the Graph Model is also examined by Sakakibara et al. [29]. This approach specifies the minimum information needed for analysts or third parties to perform stability analyses. The robustness analysis developed in this work is general enough to be applied to conflicts in which DM preferences are not fully known. This work's primary goal is not to discover DM preferences, but to allow for stability analysis of conflicts with incomplete information.

Yet a different method, the inverse Graph Model, deals with the problem of preference elicitation by identifying preference rankings that could lead to a desired resolution [30,31]. Rather than working forward from DMs, options, and preferences to conflict resolutions, this approach begins by selecting a desired resolution and determining which preferences are required to bring it about. Analysts use what information they have about the DMs' preferences and then apply the inverse approach to determine what other preference rankings are necessary for creating the desired resolution. Rather than attempting to ascertain opponent preferences based on actions, this methodology bypasses the problem entirely.

3. Problem setup

Let $\mathcal{O}_i$ denote the set of options for DM i for $i \in N$, where N is the set of DMs and $|N| = n$. An option $o_j \in \mathcal{O}_i$ is an action taken by DM i. For each option under its control, a DM may select it or not; DM i selects a strategy $g^i : \mathcal{O}_i \to \{0, 1\}$ where

$$g^i(o_j) = \begin{cases} 1 & \text{if DM } i \text{ selects option } o_j \\ 0 & \text{otherwise} \end{cases}$$