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Global dynamics of an HIV model incorporating senior male clients

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ABSTRACT

In this paper, a compartmental model for the human immunodeficiency virus (HIV) infection among female sex workers and senior male clients is formulated. The qualitative analyses are carried out in terms of the basic reproduction number R_0. The global analytical results are then obtained by employing novel techniques and methods: the disease-free equilibrium is of global asymptotic stability when a threshold $R_0 \leq 1$ is satisfied; otherwise, a unique endemic equilibrium exists and it is globally asymptotically stable. Numerical simulations are performed to illustrate and extend analytical results. It is shown that diagnosis, treatment and education have positive effect on the control of HIV transmission, but senior male clients can burden the HIV epidemic and prolong the duration of the disease outbreak.

1. Introduction

The human immunodeficiency virus (HIV) infection is rapidly increasing worldwide, and it is in part driven by the commercial sex industry. Despite an increasing condom use rate, unprotected sex remains common among commercial sex workers and their clients. A survey in China [27] suggests that 20.5% of commercial sex workers never use condoms, while only 19% of them always do. Sex workers are often willing to engage in unprotected sex if their clients pay extra. The HIV/AIDS epidemic is now spreading from the high-risk groups to the general population, resulting in a larger scale transmission and higher prevalence [21].

Mathematical models have been applied to study prevention and control strategies for the spread of HIV/AIDS in given populations. For instance, Bacaër et al. [3] developed and analyzed a mathematical model of the HIV/AIDS epidemic among injecting drug users and sex workers. Roeger et al. [20] targeted a specific population of Tuberculosis (TB) and HIV co-infections where TB is the leading cause of death among individuals infected with HIV. Zhang et al. [35], in the setting of heterosexual and homosexual populations, presented the transmission of sexually transmitted diseases (such as HIV) on bipartite scale-free graphs. HIV infection among rural-to-urban migrant workers were studied through model formulation of delay differential equations by Sigdel and McCluskey [23], and Wang [32]. Hussaini et al. [12] formulated a non-linear deterministic model to assess the role of public education program on HIV transmission and revealed that the model exhib-

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ited the phenomenon of backward bifurcation. Eaton et al. [7] compared multiple independent mathematical models in four settings: South Africa, Zambia, India and Vietnam, and assessed the potential health benefits, costs, and cost-effectiveness of various eligibility criteria for adult antiretroviral therapy (for HIV infection) and expanded treatment coverage.

However, in recent years a growing number of the elderly have contracted HIV/AIDS in developing countries [5] and even in developed countries [19]. In 2005, 483 new HIV carriers aged 60 and older were diagnosed in China, accounting for 2.2% of the total. The number surged to 3,031 in 2010, or nearly 9% of the total that year. In 2012, men aged 50 and older accounted for nearly 20% of newly reported HIV cases across China [22]; it was reported that, in 2015, 83 of the 189 people (unexpected nearly 45%) tested positive for HIV across the Rio Grande Valley, USA were 55 years old and older [19]. As of 2012, UNAIDS estimates that out of the global total of 35.3 million people living with HIV, 3.6 million are people aged 50 years or older [28]. With the population rapidly ageing, the global share of older people (aged 60 years or over) increased from 9.2% in 1990 to 11.7% in 2013 and will continue to grow, reaching 21.1% of the world population by 2050 [34]. Good economic conditions make seniors remain sexually active, and the lack of safe sex knowledge may expose an increasing number of them to HIV infection. The HIV-related stigma and discrimination among elderly men are much stronger than in other groups [29], the consequences of which may include being shunned by family, peers and the wider community, facing poor treatment, human rights erosion, and psychological damage. All these factors limit seniors' access to HIV education, testing, treatment and other services [26]. Elderly men infected with human immunodeficiency virus (HIV) are usually not diagnosed and they spread the disease without being aware. Thus, the presence of elderly men may complicate the dynamics, prevention and control of HIV/AIDS epidemic. Therefore, it is important, as well as novel, to formulate a mathematical model of differential equations targeting senior male clients, to gain insight into HIV transmission among them from a theoretical perspective. We hope our work may bring attention and voice concerns among mathematical biologists, epidemiologists and policy makers for HIV acquisition of elderly male.

The paper is structured as follows. The model is formulated in Section 2. The threshold expression for epidemic and the stability properties of disease-free equilibrium are presented in Section 3. In Section 4, the existence and the global asymptotical stability of the endemic equilibrium are investigated. The paper ends with numerical simulations and discussion to illustrate and extend the found analytical results.

2. Mathematical model

A mathematical model of HIV infection is formulated under the following assumptions:

(1) Two groups: female sex workers and senior male clients, are considered in the model.
(2) The population of female sex workers is divided into three compartments: the susceptible individuals, the undiagnosed HIV-positive individuals, and the diagnosed HIV-positive individuals. The female sex workers diagnosed with HIV may stay in the business for some reasons such as poverty, revenge for having been infected unjustifiably and lack of knowledge on disease dynamics [15]. Fortunately, the diagnosed are usually counselled and the counselling is partially effective in preventing high-risk sexual behaviors; moreover, they might receive treatment, such as antiretroviral therapy (ART), to suppress the viral load in the patient's body [18]. These factors suggest that all exposures are not equal and those exposures to the diagnosed female HIV-positive individuals carry, on average, a lower risk of HIV transmission than that to the undiagnosed ones.
(3) The population of senior male clients is divided into two compartments: the susceptible individuals, and the undiagnosed HIV-positive individuals. Considering the fact that senior males usually do not check their HIV status due to the knowledge lack on HIV/AIDS and/or the external stigma and discrimination they may face, the diagnosed HIV-positive individuals for senior male clients are ignored in the model.
(4) The AIDS compartment is not considered in the model since full blown AIDS individuals are usually hospitalised or sexually inactive. They are assumed not to engage in HIV transmission activities, and then do not contribute to HIV infection.
(5) The total population is assumed a homogeneous mixing. This means that susceptible individuals are equally likely to be infected by an infectious individual in the case of a contact.
(6) All those recruited into the population of female sex workers and senior male clients are, respectively, assumed susceptible.
(7) The model in this paper can be applied in two scenarios. Scenario 1: consider the asymptomatic stage of HIV infection, including the 2–4-week acute HIV infection stage and the 10-year long clinical latency stage before the onset of AIDS. During the asymptotical stage, people who are infected with HIV experience no symptoms, or only mild ones. The HIV-related death during this stage is then negligible. Scenario 2: in the setting of asymptomatic and symptomatic stages of HIV infection, taking senior male clients for example, the removal rate for the susceptible (natural death rate+the rate of stopping buying sex) is assumed approximately equal to that for the infectives (natural death rate+disease-caused death rate+ the rate becoming AIDS patients)

The notation and implication of five variables are listed below:
S_m: number of susceptible senior male clients;
I_m: number of undiagnosed HIV-positive senior male clients;
S_f: number of susceptible female sex workers;