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A new collocation approach for solving systems of high-order linear Volterra integro-differential equations with variable coefficients

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ABSTRACT

This paper contributes an efficient numerical approach for solving the systems of high-order linear Volterra integro-differential equations with variable coefficients under the mixed conditions. The method we have used consists of reducing the problem to a matrix equation which corresponds to a system of linear algebraic equations. The obtained matrix equation is based on the matrix forms of Fibonacci polynomials and their derivatives by means of collocations. In addition, the method is presented with error. Numerical results with comparisons are given to demonstrate the applicability, efficiency and accuracy of the proposed method. The results of the examples indicated that the method is simple and effective, and could provide an approximate solution with high accuracy or exact solution of the system of high-order linear Volterra integro-differential equations.

1. Introduction

Integral and integro-differential equations are a well-known mathematical tool for representing physical problems. Historically, they have achieved great popularity among mathematicians and physicists in formulating boundary value problems of gravitation, electrostatics, fluid dynamics and scattering. It is also well known that initial-value and boundary-value problems for differential equations can often be converted into integral equations and there are usually significant advantages to be gained from making use of this conversion. Among these equations, Fredholm and Volterra integro-differential equations or systems of these equations arise from various applications, like glass-forming process [1], nano-hydrodynamics [2], drop wise condensation [3], examining the noise term phenomenon [4], modeling the competition between tumor cells and the immune system [5], and wind ripple in the desert [6].

Since few of integro-differential equations systems can be solved explicitly, it is often necessary to find the numerical techniques for solving such systems. Recently, systems of the integral and integro-differential equations have been solved using the Adomian decomposition methods [7], the Lagrange method [8], the rationalized Haar functions method [9,10], the Legendre matrix method [11], the Bessel matrix method [12], the homotopy perturbation method [13,14], the modified homotopy perturbation method [15], the Galerkin method [16], the variational iteration method [17], the Chebyshev polynomial method [18], the Tau method [19], the differential transform method [20], the Runge–Kutta methods [21], the spline

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approximation method [22], the block pulse functions method [23], the spectral method [24], the differential transform method [25], the finite difference approximation method [26] and delta basis function method [27].

Our aim in this paper is finding an approximate solution for system of high order linear Volterra integro-differential equations with variable coefficients by means of the Fibonacci collocation method that was used for solving Volterra–Fredholm integral equations in [28].

One generalization of the Fibonacci numbers is the sequence of Fibonacci polynomials

$$F_n(t) = tF_{n-1}(t) + F_{n-2}(t), \qquad n \geq 2,$$

with initial conditions $F_0(t) = 0,\ F_1(t) = 1$. From these expressions, we can write

$$F_{n+1}(t) = \sum_{i=0}^{\lfloor \frac{n}{2} \rfloor} \binom{n-i}{i} t^{n-2i}, \quad n \geq 0,$$

where $\lfloor t \rfloor$ denotes the greatest integer in t.

Let $\mathcal{T}(t) = [1, t, t^2, t^3, \dots, t^N]^T$ and $\mathcal{F}(t) = [F_1(t), F_2(t), F_3(t), \dots, F_{N+1}(t)]^T$. So the equations for the Fibonacci polynomials can be written in the matrix form

$$\mathcal{F}(t) = \Gamma\ \mathcal{T}(t), \tag{1}$$

where Γ is the lower triangular matrix with entrances the coefficients appearing in the expansion of the Fibonacci polynomials in increasing powers of t that is invertible [29].

Furthermore, the dual matrix of $\mathcal{F}(t)$ is defined as follows:

$$\mathcal{Q}(t) = \int_a^t \mathcal{F}(s)\mathcal{F}^T(s)ds = \int_a^t \Gamma\mathcal{T}(s)\mathcal{T}^T(s)\Gamma^T ds = \Gamma\mathcal{H}(t)\Gamma^T,$$

where

$$\mathcal{H}(t) = \int_a^t \mathcal{T}^T(s)\mathcal{T}(s)ds = [h_{u,v}], \quad h_{u,v} = \frac{t^{u+v-1} - a^{u+v-1}}{u+v-1}, \ u, v = 1, 2, \dots, N+1.$$

This paper is organized as follows. In Section 2, we approximate the one and two variable function by the truncated Fibonacci series and present a numerical method for solving a system of linear Volterra integro-differential equations with variable coefficients. Convergence analysis for the method is established in Section 3. To support our findings, we present numerical results of some experiments using Matlab in Section 4. Section 5 concludes this article with a brief summary.

2. Fibonacci matrix method

Consider the system of high order linear Volterra integro-differential equations with variable coefficients

$$\sum_{m=0}^{M}\sum_{j=1}^{J} \rho_{ij}^m(t)\psi_j^{(m)}(t) - \int_a^t \sum_{j=1}^{J} k_{ij}(t,s)\psi_j(s)ds = \phi_i(t), \ \ i = 1, 2, \dots, J, a \leq t \leq b, \tag{2}$$

under the mixed conditions

$$\sum_{j=0}^{M-1} (\alpha_{ij}^m \psi_m^{(j)}(a) + \beta_{ij}^m \psi_m^{(j)}(b) + \gamma_{ij}^m \psi_m^{(j)}(c)) = \lambda_{mi}, \tag{3}$$

where $i = 0, 1, \dots, M-1,\ \ m = 1, 2, \dots, J,\ \ a \leq c \leq b,\ \ \psi_j^{(0)}(t) = \psi_j(t)$ is an unknown function, the known $\rho_{ij}^m(t),\ \varphi_i(t)$ and $k_{ij}(t, s)$ are defined on the interval $a \leq t, s \leq b$ and $\alpha_{ij}^m,\ \beta_{ij}^m,\ \gamma_{ij}^m$ and $\lambda_{mi},\ i, j = 0, 1, \dots, M-1,\ n = 1, 2, \dots, J$ are appropriate constants.

Suppose that the solution of this system expressed in terms of the Fibonacci polynomials as

$$\psi_i(t) \simeq \sum_{n=1}^{N+1} a_{i,n}F_n(t), \quad i = 1, 2, \dots, J, \quad a \leq t \leq b, \tag{4}$$

where $a_{i,n},\ n = 1, 2, \dots, N+1$ are the unknown Fibonacci coefficients, N is any arbitrary positive integer such that $N \geq M$, and $F_n(t),\ n = 1, 2, \dots, N+1$ are the Fibonacci polynomials. So we have

$$\psi_j(t) \simeq \mathcal{A}_j\mathcal{F}(t), \quad j = 1, 2, \dots, J, \tag{5}$$

where $\mathcal{A}_j = [a_{j,1}, a_{j,2}, \dots, a_{j,N+1}]$. Then from Eqs. (1) and (5), the function defined in relation (4) can be written in the matrix form

$$\psi_j(t) \simeq \mathcal{A}_j\Gamma\ \mathcal{T}(t), \quad j = 1, 2, \dots, J. \tag{6}$$