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New stability results for delayed neural networks☆

Hanyong Shao*, Huanhuan Li, Chuanjie Zhu

The Research Institute of Automation, Qufu Normal University, Rizhao 276826, China

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ABSTRACT

This paper is concerned with the stability for delayed neural networks. By more fully making use of the information of the activation function, a new Lyapunov–Krasovskii functional (LKF) is constructed. Then a new integral inequality is developed, and more information of the activation function is taken into account when the derivative of the LKF is estimated. By Lyapunov stability theory, a new stability result is obtained. Finally, three examples are given to illustrate the stability result is less conservative than some recently reported ones.

1. Introduction

In the past decades, neural network (NN) has been successfully applied in signal processing, pattern recognition, associative memory, optimization problem, and other engineering and scientific areas [1,2]. However, during the implementation of artificial NNs, the finite switching speed of amplifiers and the inherent communication time between the neurons inevitably introduce time delay, which might cause oscillation, divergence, and even instability. Therefore, the stability of the neural networks with a time-varying delay (DNNs) has attracted a large number of researchers, and some stability criteria have been reported in the literature. The stability criteria developed for DNNs can be divided into delay-independent ones and delay-dependent ones. Compared to the former, the delay-dependent stability criteria, which include the information of time delay, usually have less conservative, especially when applied to DNNs with small delay. Thus, more attentions have been paid to delay-dependent stability analysis and its main goal is to reduce the conservatism of the derived stability condition.

In terms of the Lyapunov stability theory, the conservatism of the derived stability condition is related to the choosing of the LKF and dealing with its derivative. Constructing a generalized LKF is an effective way to reduce conservatism of the stability results obtained, and various types of LKF have been reported. In [3], an augmented LKF was constructed by introducing the information of the delayed state and integral terms of the state and activation function. In [4,5], LKFs were constructed based on multiple integrals, with triple integrals in [4], and quadruple integrals in [5] respectively. By decomposing delay interval into segments, delay-partitioning based LKFs were constructed in [6,7] while an activation function based LKF was constructed in [8]. To deal with the derivative of LKF, firstly introducing slack variables is an important way to take more information of neural networks and derive less conservative stability results [9–11]. Secondly advanced bounding techniques are necessary when estimating the derivative of LKF. In order to derive tighter upper bound of the derivative of LKF, numerous inequalities have been proposed. The Jensen inequality was employed in [12,13], and the Wirtinger-based inequality was used in [14]. Thirdly, to check the negative definite of the derivative of LKF, some methods were needed.

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* Corresponding author.
E-mail address: hanyongshao@163.com (H. Shao).

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The convex combination technique, for example, was adopted in [15–17], where the convex polyhedron method was used in [16] and the quadratic convex combination technique was employed in [17].

Recently stability results for neural networks have been reported in the literature. In [18], asymptotic stability criterion was obtained using a LKF including a triple integral, where the Wirtinger-based inequality was employed to estimate the derivative of the LKF. In [19], by defining a more general LKF, a delay-dependent stability result was formulated in linear matrix inequality, while a combined convex approach to stability for DNN was studied in [20]. Very recently stability analysis was conducted in [21] and a new stability result was derived, where the tradeoff between conservatism and complexity was considered. We can see each of those papers has its characteristics, in terms of the LKF or the estimating approach for the derivative of the LKF. However, there is still room for those papers to improve.

In this paper, attention is focused on revisiting the stability analysis problem for general DNNs. A new LKF with more fully making use of the information of the activation function is defined. When estimating the derivative of the LKF, a new integral inequality and more information of the activation function are introduced. Based on Lyapunov stability theory, a new delay-dependent stability criterion is formulated in terms of linear matrix inequality. Examples are listed to illustrate the reduced conservatism of the stability result.

Notations: Throughout this paper $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, and $\mathbb{R}^{n\times m}$ is the set of all $n \times m$ real matrices; the superscript 'T' and '-1' stand for the transpose and inverse of a matrix, respectively; 'I' and '0' represent the identity and null matrices with appropriate dimensions, respectively; the notation $|\cdot|$ denotes the absolute value; the notation $diag\{\cdots\}$ stands for a block-diagonal matrix; the notation $P > 0(\geq 0)$ means that P is a real symmetric and positive-definite (semipositive-definite) matrix. Moreover, for any square matrix A, we define $Sym\{A\} = A + A^T$, and the symmetric term in the matrix is denoted by $*$.

2. Problem formulation

Consider the general DNN with a time-varying delay $\tau(t)$ [21,22]:

$$\dot{u}(t) = -Au(t) + W_0 g(Wu(t)) + W_1 g(Wu(t-\tau(t))) + J \tag{1}$$

where $u(t) = [u_1(t), u_2(t), \ldots, u_n(t)]^T$ is the state vector associated with the n neurons; $g(\cdot) = [g_1(\cdot), g_2(\cdot), \ldots, g_n(\cdot)]^T$ represents the neuron activation function with $g(0) = 0$; $A = diag\{a_1, a_2, \ldots, a_n\} > 0$; W, W_0 and W_1 are the connection weight matrices; $J = [J_1, J_2, \ldots, J_n]^T$ is a vector representing the bias; and $\tau(t)$ is a time-varying delay satisfying

$$0 \leq \tau(t) \leq h, \quad \dot{\tau}(t) \leq \mu \tag{2}$$

The following assumption is made throughout this paper.

Assumption 1. The neuron activation function is bounded, and satisfies

$$l_i^- \leq \frac{g_i(s_1) - g_i(s_2)}{s_1 - s_2} \leq l_i^+, s_1 \neq s_2, i = 1, 2, \ldots, n$$

where l_i^- and l_i^+ are known real constants.

Based on Assumption 1, there exists an equilibrium point u^* for (1), i.e.,

$$0 = -Au^* + W_0 g(Wu^*) + W_1 g(Wu^*) + J$$

To transfer the equilibrium u^* to the origin, we make the transformation $x(t) = u(t) - u^*$ to neural network (1). Then, it becomes

$$\dot{x}(t) = -Ax(t) + W_0 f(Wx(t)) + W_1 f(Wx(t-\tau(t))) \tag{3}$$

where $x(t) = [x_1(t), x_2(t), \cdots, x_n(t)]^T$ is the state vector of the transformed system (3), $f(\cdot) = [f_1(\cdot), f_2(\cdot), \ldots, f_n(\cdot)]^T$ and $f_i(\bar{W}_i x(t)) = g_i(\bar{W}_i x(t) + \bar{W}_i u^*) - g_i(\bar{W}_i u^*)$ with $f_i(0) = 0$ and $\bar{W}_i$ denoting the ith row vector of the matrix W. It is noted that

$$l_i^- \leq \frac{f_i(s_1) - f_i(s_2)}{s_1 - s_2} \leq l_i^+, s_1 \neq s_2 \tag{4}$$

which implies

$$l_i^- \leq \frac{f_i(s)}{s} \leq l_i^+, s \neq 0$$

This paper aims to derive a delay-dependent stability criterion of DNN (3) with (2) and (4) to determine the admissible upper bound of $\tau(t)$, which can guarantee the stability of the DNN. To the end, we need the following lemmas:

Lemma 1 (Jensen's Inequality [23]). *For any matrix $R \in \mathbb{R}^{n\times n}$, $R = R^T > 0$, scalars $\alpha < \beta$, and vector $x : [\alpha, \beta] \to \mathbb{R}^n$, such that the integration concerned is well defined, then*

$$(\beta - \alpha)\int_\alpha^\beta x^T(s)Rx(s)ds \geq \int_\alpha^\beta x^T(s)ds R \int_\alpha^\beta x(s)ds$$