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Continuous dependence for a thermal convection model with temperature-dependent solubility☆

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We study the structural stability for a thermal convection model with temperature-dependent solubility. When the spatial domain Ω is bounded in $\mathbb{R}^3$, we show that the solution depends continuously on the Boussinesq coefficient λ by using the method of a second order differential inequality. In the procedure of deriving the result, we also get the a priori bounds for the temperature T and the salt concentration C.

1. Introduction

The question of continuous dependence of solutions in partial differential equations has been extensively studied in recent years. Sometimes we refer it as the question of structural stability. The concept of structural stability is the study of continuous dependence on changes in the model itself rather than the initial data. Many references to work of this nature were given in the monograph of Ames and Straughan [1], which studied the structural stability with respect to changes in the model itself. This means changes in coefficients in the partial differential equations can be reflected physically by changes in constitutive parameters. The mathematical analysis of these equations will help us to reveal their applicability in physics. On the other hand, continuous dependence results are important because of the inevitable error that arises in both numerical computation and the physical measurement of data. It is relevant to know the magnitude of the effect of such errors in the solutions. We believe it is valuable to study the subject of structural stability.

There are many papers in literature dealing with the structural stability for varieties equations. Most of them focused on the Brinkman, Darcy and Forchheimer models. These equations were discussed in the books of Nield and Beijan [17] and Straughan [25,26]. Several papers in the literature dealt with Saint-Venant type spatial decay results for Brinkman, Darcy, Forchheimer and other equations for porous media. More recent work on stability and continuous dependence questions in porous media problems has been carried out by Ames and Payne [2], Franchi and Straughan [7], Kaloni and Qin [11], Kaloni and Guo [12], Payne and Straughan [19–21], Payne, Song and Straughan [22], Lin and Payne [14–16], Li and Lin [13], Celebi et al. [3,4], Straughan [27], Scott [23], Scott and Straughan [24], and Harfash [8–10].

The fundamental model we study is based upon the equations of balance of momentum, balance of mass, conservation of energy, and conservation of salt concentration (see [5]).

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$$\begin{cases} \frac{\partial u_i}{\partial t} + \lambda u_j \frac{\partial u_i}{\partial x_j} = -p_{,i} + \Delta u_i + g_i T - h_i C & (x,t) \in \Omega \times [0,\tau], \\ \frac{\partial u_i}{\partial x_i} = 0 \quad (x,t) \in \Omega \times [0,\tau], \\ \frac{\partial T}{\partial t} + u_i \frac{\partial T}{\partial x_i} = \Delta T \quad (x,t) \in \Omega \times [0,\tau] \\ \frac{\partial C}{\partial t} + u_i \frac{\partial C}{\partial x_i} = \Delta C + Lf(T) - kC \quad (x,t) \in \Omega \times [0,\tau], \end{cases} \tag{1.1}$$

where u_i is the velocity, p denotes the pressure, T is the the temperature and C is the salt concentration. Here $g_i(x)$, $h_i(x)$ are gravity fields. Without loss of generality, we assume g_i, h_i satisfy $|g| \leq 1$ and $|h| \leq 1$. Here also Δ is the Laplacian operator. a, b, L and k are positive constants. Eq. (1.1) follow in practice by employing a Boussinesq approximation which accounts for variable C allowing the incompressibility condition to hold, cf. Fife [6, pp. 72–74]. The function f is at least a C^1 function.

Eq. (1.1)hold in the region $\Omega \times [0, \tau]$, where Ω is a bounded, simply connected and star-shaped domain with boundary $\partial\Omega$ in $\mathbb{R}^3$, and τ is a given number which satisfies $0 \leq \tau < \infty$. Associated with (1.1), we impose the boundary conditions

$$u_i = 0, \quad T = g(x,t), \quad C = h(x,t) \quad (x,t) \in \partial\Omega \times [0,\tau], \tag{1.2}$$

and additionally the concentration is given at $t = 0$, i.e.,

$$u_i(x,0) = u_{i0}(x), \quad T(x,0) = T_0(x), \quad C(x,0) = C_0(x) \quad x \in \Omega. \tag{1.3}$$

In [5], the authors obtained the continuous dependence result on the reaction coefficients for Eq. (1.1) in $\mathbb{R}^2$. For the case in $\mathbb{R}^3$, the Sobolev inequality is no longer applicable. This inequality is an important tool for proving the results obtained in [5]. We will derive the continuous dependence result on the Boussinesq coefficient λ in $\mathbb{R}^3$. We cannot follow the method presented in [5], because the case when $\Omega \subset \mathbb{R}^3$ is more difficult to tackle than the case in [5]. We can get the structural stability result for the Boussinesq equations in $\Omega \subset \mathbb{R}^3$. I have never seen such a result for these equations. What is more, the problem we studied in this paper has the same nonlinear item $u_j \frac{\partial u_i}{\partial x_j}$ as the Navier–Stokes equation, so the method proposed in this paper can be useful in studying the property of solution of the Navier–Stokes equation. In these senses, our paper is interesting and involved.

The plan of this paper is as follows. In the next section, we derive a priori bounds for the temperature T and the salt concentration C which are very useful in deriving our result. In Section 3, we study the continuous dependence of a solution to the thermal convection model on the Bounssinesq coefficient. Using the technique of a second order differential inequality, we can get the desired continuous dependence result. Some conclusions are included in Section 4.

In the present paper, the comma is used to indicate partial differentiation and the differentiation with respect to the direction x_k is denoted as , k, thus $u_{,i}$ denotes $\frac{\partial u}{\partial x_i}$. The usual summation convection is employed with repeated Latin subscripts summed from 1 to 3. Hence, $u_{i,i} = \sum_{i=1}^{3} \frac{\partial u_i}{\partial x_i}$, $\|\cdot\|$ denotes the norm of L^2, and $\|\cdot\|_p$ denotes the norm of L^p.

2. A priori bounds for the temperature *T* and the salt concentration *C*

In the course of producing the result of continuous dependence on the coefficient of (1.1), we find it is easy if we can derive a priori bounds for the temperature T and the salt concentration C.

In [18, pp. 432–433], Payne, Rodrigues and Straughan reached the result

$$\sup_{[0,\tau]} \|T\|_\infty \leq T_M, \tag{2.1}$$

where

$$T_M = \max\left\{ \|T_0\|_\infty, \sup_{[0,\tau]} g_\infty \right\}$$

and g_∞ is the maximum of g on $\partial\Omega$.

In order to get a bound for the salt concentration C, we divide C into $C = C_1 + C_2$, where C_1 and C_2 satisfy the following equations respectively:

$$\begin{cases} \frac{\partial C_1}{\partial t} + u_i \frac{\partial C_1}{\partial x_i} = \Delta C_1 - kC_1, \\ C_1(x,t) = h(x,t) \quad (x,t) \in \partial\Omega \times [0,\tau], \\ C_1(x,0) = C_0(x) \quad x \in \Omega, \end{cases} \tag{2.2}$$

and

$$\begin{cases} \frac{\partial C_2}{\partial t} + u_i \frac{\partial C_2}{\partial x_i} = \Delta C_2 + Lf(T) - kC_2, \\ C_2(x,t) = 0 \quad (x,t) \in \partial\Omega \times [0,\tau], \\ C_2(x,0) = 0 \quad x \in \Omega. \end{cases} \tag{2.3}$$