



Nonlocal interaction driven pattern formation in a prey–predator model



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ABSTRACT

A widely observed scenario in ecological systems is that populations interact not only with those living in the same spatial location but also with those in spatially adjacent locations, a phenomenon called nonlocal interaction. In this paper, we explore the role of nonlocal interaction in the emergence of spatial patterns in a prey–predator model under the reaction–diffusion framework, which is described by two coupled integro–differential equations. We first prove the existence and uniqueness of the global solution by means of the contraction mapping theory and then conduct stability analysis of the positive equilibrium. We find that nonlocal interaction can induce Turing bifurcation and drive the formation of stationary spatial patterns. Finally we carry out numerical simulations to demonstrate our analytical findings.

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1. Introduction

The theory of pattern formation dates back to the pioneering work by Turing [1] in 1952, who studied a reaction–diffusion model consisting of two chemical species, an activator and an inhibitor. The activator stimulates the production of the other species, which, in turn, inhibits the generation of the activator. Turing found that as time advances the concentrations of the two species can evolve from an initial near-homogeneity into a spatially inhomogeneous pattern when the diffusion of the inhibitor is greater than that of the activator. This finding implies that the equilibrium of the nonlinear system, which is stable in the absence of diffusion, can be destabilised by diffusion, a counter-intuitive observation, as diffusion is usually considered to exhibit a stabilising effect. This scenario is termed as Turing instability and the emergent spatial patterns are called Turing patterns. Since Turing's work, pattern formation has attracted great attention from both theorists and experimentalists during the last few decades. Levin and Segel [2] for the first time extended Turing idea from chemistry to ecology. They added diffusion to a planktonic system and theoretically investigated how diffusion leads to emergence of spatial patterns. Their method turns out to be a routine framework for studying pattern formation.

Much research effort has been devoted to the mechanisms behind the emergence of spatial patterns and a variety of mechanisms have been found. These mechanisms can basically be classified into two categories. The first category is that spatial patterns emerge from locally stable kinetic dynamics due to the presence of self-diffusion [1–4], cross-diffusion [5–7], and advection [8–10]. The other category is that spatial patterns emerge from locally unstable kinetic dynamics (such as limit

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cycle, chaos) through spatially inhomogeneous perturbations. Thus, factors that can destabilise the local kinetic dynamics can potentially be mechanisms for pattern formation, for instance, time delay [11–13] and single generation cycles [14]. It is worth pointing out that there is generally a requirement on the diffusion rates of involving species to trigger pattern formation in the former category but not in the latter [15]. Recently, population dynamics with free boundary (i.e., size of spatial domain can change as time advances) in a heterogeneous environment have been considered and it was shown that the free boundary can also affect the spatial distributions of population [16,17].

A common feature of above mentioned works is that a given individual interacts only with those living in the same spatial location (i.e., local interaction). In reality interactions can possibly occur within individuals living in adjacent locations (i.e., nonlocal interaction), which has been receiving growing attentions [18–20]. Moreover, some biological experiments demonstrated that bacteria cultures in Petri dishes release toxic substances or nutrients, which cause nonlocal interactions [21,22]. In contrast to local interactions where spatial and temporal scales of movements and interactions are comparable, nonlocal interactions assume a large spatial scale of movement but a small temporal scale of movement compared to other processes. Nevertheless, the difference between spatial and temporal scales is not too big to be indistinguishable [23]. The biological mechanism of the nonlocal interactions can be attributed to the effect of mobility [18]. When predators hunt sparse preys, their motion direction should depend not only on the prey population at the same spatial location but also on some weighted average of the population from other locations [18].

Recently researchers have considered spatio-temporal models of interacting populations (such as preypredator and competition models) with nonlocal interactions and investigated the existence of traveling wave, periodic traveling wave and modulated traveling wave [19,20,24–26]. In [27], Tanzy et al. considered in a competition model the effect of asymmetric kernel function of nonlocal interaction on the resulting patterns. In [28,29] the authors studied in a predator–prey model the effect of nonlocal interaction on the oscillatory and chaotic patterns as well as the existence of multiple stationary branches, in particular, the transition from one type of pattern to another. However, all of these studies focused on one-dimensional spatial domain, which raises a question to which degree the achieved results apply to higher dimensional space. More importantly, we wonder whether the emergent spatial patterns whenever feasible are solely due to the nonlocal interactions or combined effects of nonlocal interactions with other factors such as alternative spatially inhomogeneous steady state and/or Hopf bifurcation.

Motivated by these questions, we consider the following prey–predator model

$$\begin{cases} \partial u_1/\partial t - d_1 \Delta u_1 = u_1(a_1 - b_{11}\phi_\delta * u_1 - b_{12}\phi_\delta * u_2), & t > 0, \quad x \in \Omega, \\ \partial u_2/\partial t - d_2 \Delta u_2 = u_2(-a_2 + b_{21}\phi_\delta * u_1 - b_{22}\phi_\delta * u_2), & t > 0, \quad x \in \Omega, \\ \partial u_1/\partial \nu = \partial u_2/\partial \nu = 0, & t > 0, \quad x \in \partial\Omega, \\ u_1(0, x) = u_{10}(x), \quad u_2(0, x) = u_{20}(x), & x \in \Omega. \end{cases} \quad (1.1)$$

Here u_1 and u_2 are population densities of the prey and predator at time t and space x , respectively. Ω is a bounded domain in \mathbf{R}^n with a smooth boundary $\partial\Omega$, and ν is the outward unit normal on $\partial\Omega$, with a zero-flux on the boundaries. d_i ($i = 1, 2$) are the diffusive rates of prey and predator, respectively. a_1 is the intrinsic growth rate of the prey while a_2 the background death rate of predator. b_{ii} ($i = 1, 2$) are parameters related to nonlocal intraspecific competition, while b_{12} and b_{21} are parameters related to nonlocal predation. The nonlocal interactions are given by the presentation of convolution between a spatial kernel function ϕ_δ and population density. The spatial kernel function measures the extent to which a given individual can interact with others at different spatial locations. For instance, the effect of predation on per capital growth rate of prey is given by the term $-b_{12}u_1\phi_\delta * u_2$, while the contribution of prey on the per capital growth rate of predator is given by the term $b_{21}u_2\phi_\delta * u_1$. All above appearing parameters are positive, and we consider positive initial conditions, that is, $u_{i0}(x) \geq 0$ for $i, j = 1, 2$. Moreover, in order to ensure the existence of the classical solution, we assume that $u_{i0}(x) \in C^\alpha(\Omega)$.

The convolution terms in (1.1) are mathematically represented as

$$\phi_\delta * u_i = \int_{\mathbf{R}^n} \phi_\delta(x-y)u_i(t, y)dy, \quad (1.2)$$

where ϕ_δ is a spatial kernel function satisfying

$$\int_{\mathbf{R}^n} \phi_\delta(y)dy = 1. \quad (1.3)$$

We also assume that ϕ_δ is an even, nonnegative function with support in the interval $[-\delta, \delta]^n$. These conditions on ϕ_δ imply that ϕ_δ approaches a δ -function as $\delta \rightarrow 0$. Thus, the interactions turn out to be local in this limit. Several different kernel functions have been studied in the literature. The simplest one is the step function [30–32], which is symmetric, unweighted average over a specified interval, $\phi_\delta(x) = \frac{1}{2\delta}, x \in (-\delta, \delta)$, whilst $\phi_\delta(x) = 0, x \notin (-\delta, \delta)$. A function with a continuous, rather than abrupt, transition from unity to zero is also possible, but if the transition is sufficiently rapid, the transition region can be neglected, such as the kernel function parabolic segments [18] $\phi_\delta(x) = \frac{3}{4\delta^2}(\delta^2 - x^2), x \in (-\delta, \delta)$, whilst $\phi_\delta(x) = 0, x \notin (-\delta, \delta)$, and elevated triangle [32] $\phi_\delta(x) = \frac{\alpha\delta - |x|}{(2\alpha - 1)\delta^2}, x \in (-\delta, \delta)$, whilst $\phi_\delta(x) = 0, x \notin (-\delta, \delta)$.

The paper is structured as follows. We prove the existence and uniqueness of the solution to the problem (1.1) in the next section. The difficulty is to raise the regularity of the solutions to (1.1) including the spatial convolution. To overcome

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