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A note on the nonuniqueness of the Equal Profit Method

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ABSTRACT

When a set of players cooperate, they need to decide how the collective cost should be allocated among them. Cooperative game theory provides several methods or solution concepts, that can be used as a tool for cost allocation. In this note, we consider a specific solution concept called the Equal Profit Method (EPM). In some cases, a solution according to the EPM is any one of infinitely many solutions. That is, it is not always unique. This leads to a lack of clarity in the characterization of the solutions obtained by the EPM. We present a modified version of the EPM, which unlike its precursor ensures a unique solution. In order to illustrate the differences, we present some numerical examples and comparisons between the two concepts.

1. Introduction

In applications where competitive players cooperate, fairness is a key to maintaining the cooperation. Reduced collective cost may be an incentive to cooperate, and it is important that the costs, or cost reductions, that are allocated to the players meet certain fairness criteria. Two distinct solutions which are equally acceptable according to a solution concept or fair according to certain fairness criteria, may still have a dire consequence on individual players. That is, a player may be allocated a higher cost due to arbitrariness. In order to prevent an arbitrary choice, it is desirable to find a unique solution.

The Equal Profit Method (EPM) was introduced in [1] in an application of collaborative transportation in the forest industry. It is based on the fact that in a negotiation situation, it is advantageous to argue that all cooperating freight forwarders have an equal, or as similar as possible, relative cost, e.g. the allocated cost should be proportional to each player's stand-alone cost. As a consequence of its definition (see Section 1.2), the EPM does not guarantee a unique solution. For the sake of fairness, it is therefore important to introduce a procedure to find a unique solution according to the EPM, while remaining loyal to its overall aim. We call the modified version of the EPM, including this procedure, the Lexicographic Equal Profit Method (EPML).

With respect to the aim of the EPM, namely to minimize the differences in relative costs, the procedure to find a unique solution according to the EPM is inspired by an iterative process of finding the Nucleolus of a cooperative game (see [2,3]) and uses the same technique. *Lexicography* (see Section 1.3) has a major role in this procedure which is described in Section 2. The procedure terminates when a unique solution is found, hence it is guaranteed to be a unique solution, assuming at least one solution exists.

Game theory and cooperative game theory are used in different engineering applications. We refer to [4–6] for surveys and overviews.

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The EPM has been studied in the context of cooperation in logistics where costs, emissions or utility is to be allocated between players. Some recent papers are the following.

A number of solution concepts are tested in [7] when the transportation cost of forest fuel is reduced due to cooperation. One of the solution concepts is a version of the EPM where the group rationality (see Section 1.1) is relaxed. The potential cost reduction, based on the data used, is 22%.

The CO_2 emissions caused by each customer of a logistics service provider may be estimated by allocation methods according to Naber et al. [8]. They compare five solution concepts on a case study and a number of randomly generated instances. The non-uniqueness of the EPM is pointed out, which may affect the robustness and consistency. They suggest further investigation, which is a motivation for this note.

Utility is allocated between the different owners of a network in [9]. A solution concept is suggested that is motivated by the EPM, the Equal Utility Method. The network is a multi-owner transshipment and logistics network containing elements of uncertainty and is modeled as a maximum-flow problem.

The EPM is used in [10] when a cost-sharing problem arises among service providers. A service capacity pooling system is studied. The EPM provides reasonable cost-sharing results based on the specific problem. The stability of the coalition is maintained.

A number of solution concepts are compared in [11] when studying horizontal cooperation between logistics service providers. The logistics service providers may share a number of distribution centers or warehouses. The test instances are based on a case study in the UK.

Game theory is related to dynamical processes. Dynamic systems are studied in many applications, e.g. [12].

Before defining the EPML, we present below some basics of cooperative cost games, the EPM and the procedure for finding the Nucleolus.

1.1. Cooperative cost games

A cooperative cost game is defined by a pair (N, c) where N is a set of n players and c is a function defined $\forall S \subseteq N$, mapped on $\mathbb{R}$. The value $c(S)$ is interpreted as the cost of a cooperation between the players in S, and naturally $c(\emptyset) := 0$. We call S a *coalition*, N the *grand coalition* and the function c the *characteristic function*. A *solution* is a vector $y \in \mathbb{R}^n$ where y_i represents the cost allocated to player i. For simplicity, we write $c(i) := c(\{i\})$ and $y(S) := \sum_{i \in S} y_i$. A *solution concept* yields a set, Y, of solution candidates for a given cooperative cost game (N, c). A *solution according to a solution concept* is understood to be any element $y \in Y$. If, for a given solution concept and some cooperative cost game, Y is infinite or $|Y| > 1$ for finite sets, we say that the solution is *non-unique* or that the solution concept has the property of non-uniqueness.

1.2. Equal Profit Method

Before defining the EPM we define the core of a cooperative game, which is an essential part of the EPM. The core was introduced in [13] and is based on two statements: the total cost should be allocated among the players and no coalition should be allocated a higher cost in the grand coalition compared to if the coalition acted alone. Mathematically this is formulated as:

$$y(N) = c(N), \tag{1a}$$

$$y(S) \leqslant c(S), \quad \forall S \subset N. \tag{1b}$$

If a solution, y, satisfies these constraints, we say that it is a *stable solution*, or equivalently, that the solution is in the core. The constraint (1a) is called *efficiency* and the constraint set (1b) is called *group rationality*. If a solution satisfies the constraints in (1b) for all singleton coalitions, that is $|S| = 1$, then y satisfies *individual rationality*.

When the EPM is considered, costs are allocated in such a way the relative costs, $\frac{y_i}{c(i)}$, are as equal as possible and ideally equal, for all players. Furthermore, a solution according to the EPM must be stable.

Given a cooperative cost game (N, c), the set of solution candidates yielded by the EPM is the set of all optimal solutions to the linear programing problem P_0.

$$P_0 \quad \min f_0, \tag{2a}$$

$$s.t. \quad \frac{y_i}{c(i)} - \frac{y_j}{c(j)} \leqslant f_0, \quad \forall (i, j) \in N \times N, \tag{2b}$$

$$y(S) \leqslant c(S), \quad \forall S \subset N, \tag{2c}$$

$$y(N) = c(N), \tag{2d}$$

where $\frac{y_i}{c(i)} - \frac{y_j}{c(j)}$ denotes the difference in relative costs between two players, i and j. The variable f_0 in the constraint set (2b) represents the largest difference in relative costs. The constraint set (2c) and constraint (2d) are simply the core