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Short Communication

On the edge-version atom-bond connectivity and geometric arithmetic indices of certain graph operations

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ABSTRACT

Let $d_{L(G)}(e)$ be the degree of an edge e in line graph $L(G)$ of a graph G. The edge versions of atom-bond connectivity (ABC_e) and geometric arithmetic (GA_e) indices of G are defined as $\sum_{ef\in E(L(G))}\sqrt{\frac{(d_{L(G)}(e)+d_{L(G)}(f)-2}{d_{L(G)}(e)\times d_{L(G)}(f)}}$ and $GA_e(G)=\sum_{ef\in E(L(G))}\frac{2\sqrt{d_{L(G)}(e)d_{L(G)}(f)}}{d_{L(G)}(e)+d_{L(G)}(f)}$. In this paper, we study the ABC_e and GA_e indices for joint graphs and certain graph operations.

1. Introduction

As the concepts in theoretical and applied areas, a graph is represented by a collection of connecting points and lines, who can be separately called vertices and edges. Suppose e is an edge of G, which connects the vertices u and v, then we denote $e = uv$ and state that "u and v are adjacent". We can see that there is a path between every pair of vertices existing in a connected graph described above. Then the length of a shortest path between u and v in G can be determined by the distance $d(u, v)$ of two vertices u and v. A simple graph is defined as an unweighted, undirected graph who has no loops and multiple edges attached. Every individual number used to characterize some properties of a graph is known as a topological index of a related (molecular) graph. It's apparent that the numbers of vertices and edges are topological variants. The first graph invariant reported as (distance based) topological index, Wiener index, is denoted as a half of distances between all the pairs of vertices in a graph [1]. Meanwhile, on the basis of the distance between edges, the edge version of Wiener index was proposed by Iranmanesh et al. [2]. Several recent results on various types of Wiener indices (such as Wiener polarity index) can be referred to Ma et al. [3,4].

The degree of arbitrary vertex v is denoted as the number of vertices which is linked to v. There are some famous degree-based indices, which are introduced and applied in chemical engineering (for instance, Randić index, see Li and Shi [5] and Shi [6] for more details). In a graph G, if the corresponding edges share a vertex in G, the line graph $L(G)$ of a graph G is considered as a graph with vertices of the edges in G, and it possesses two adjacent vertices. Similarly, the degree of an edge $e \in E(G)$ is represented by the number of its adjacent vertices in $V(L(G))$. Estrada et al. [7] put forward a topological

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index with the name of the atom-bond connectivity index (briefly, ABC) as

$$ABC(G) = \sum_{uv \in E(G)} \sqrt{\frac{d_G(u) + d_G(v) - 2}{d_G(u) \times d_G(v)}},$$

where d_u and d_v represent the degrees of the vertices u and v, respectively. Recent advances on ABC index can be referred to Das et al. [8], Lin et al. [9], Gao and Shao [10], and Bianchi et al. [11]. Referring to the end vertex degree d_e and d_f of edges e and f in a line graph of G, Farahani [12] proposed the edge version of atom-bond connectivity index. This idea is expressed as follows:

$$ABC_e(G) = \sum_{ef \in E(L(G))} \sqrt{\frac{d_{L(G)}(e) + d_{L(G)}(f) - 2}{d_{L(G)}(e) \times d_{L(G)}(f)}},$$

where $d_{L(G)}(e)$ is the degree of the edge e in $L(G)$.

Furthermore, Randić [24] raised one of the most important topological indices as branching index, that is, the sum of certain bond contributions calculated from the vertex degree of the hydrogen suppressed molecular graphs.

With the reference to Randić connectivity index, Vukicevic and Furtula [13] put forward the geometric-arithmetic index (briefly, GA), a topological index, and it is designed on the ground of the end-vertex degrees of edges in a graph connected G with the vertex set $V(G)$ and the edge set $E(G)$. To be specific, this index can be described as

$$GA(G) = \sum_{uv \in E(G)} \frac{2\sqrt{d_G(u) d_G(v)}}{d_G(u) + d_G(v)},$$

where $d_G(u)$ states the degree of the vertex u in G. If more information about geometric-arithmetic index is wanted, please refer to Fath-Tabar et al. [14] and Yuan et al. [15].

Iranmanesh et al. [16] proposed the edge version of geometric arithmetic index, referring to the end-vertex degrees of edges in a line graph of G. This idea is described in the following:

$$GA_e(G) = \sum_{ef \in E(L(G)} \frac{2\sqrt{d_{L(G)}(e) d_{L(G)}(f)}}{d_{L(G)}(e) + d_{L(G)}(f)},$$

where $d_{L(G)}(e)$ represents the degree of the edge e in G. The paper aims to push one more step in the study of the ABC_e and GA_e indices for the joint graphs.

If more results on topological index computation are in need, please refer to Gao and Farahani [17], Vukičević and Furtula [13], Gao and Wang [18] and [19], Asadpour [20] and [13,21–26].

Joint structure and graph operation of basic molecular structures are frequently found in the new chemical compounds, nanomaterials and drugs in the fields of chemical and pharmaceutical engineering. The phenomenon provides us some hints on the significance and feasibility of the research on the chemical and pharmacological properties of these molecular structures. As for this paper, the ABC_e and GA_e indices for joint graphs and certain graph operations are determined.

2. Main results and proofs

To begin with our proofs, we present some examples of the ABC_e and GA_e indices for certain simple graphs P_n, S_n, K_n, $K_{m,n}$, W_n and C_n, and they are a path, a star, a complete graph, a complete bipartite graph, a wheel graph and a cycle, respectively.

Example 2.1 Mahimiani et al. [16]**.** The edge GA index of certain graphs P_n, S_n, K_n, $K_{m,n}$, W_n and C_n are

$$GA_e(P_n) = GA(P_{n-1}) = \frac{4\sqrt{2}}{3} + (n-4),$$

$$GA_e(S_n) = GA(K_{n-1}) = \frac{(n-1)(n-2)}{2},$$

$$GA_e(K_n) = \frac{n(n-1)^2}{2},$$

$$GA_e(K_{m,n}) = \frac{mn(m+n)}{2},$$

$$GA_e(W_n) = (n-1)\left(1 + \frac{8\sqrt{n}}{4+n} + \frac{n-2}{2}\right),$$

$$GA_e(C_n) = GA(C_n) = n.$$