Applied Mathematics and Computation 303 (2017) 190–202

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Windows for escaping particles in quartic galactic potentials

Juan F. Navarro

Department of Applied Mathematics, University of Alicante, Carretera San Vicente del Raspeig, s/n, 03690 San Vicente, Alicante, Spain

ARTICLE INFO

Keywords:
Galactic potentials
Periodic orbits
Escapes
Hamiltonian systems

ABSTRACT

We investigate the shape of the windows through which stars may escape from a galaxy modeled by a bi–symmetrical potential made up of a two–dimensional harmonic oscillator with quartic perturbing terms. The escape from the potential well is governed by the unstable periodic orbits in the openings of the potential. The unstable and stable manifolds to these periodic orbits reveal the way test particles escape from the potential well. Our main objective is to compute accurately these manifolds to analyze the shapes and sizes of the windows of escape, founding that they consist of a "main window" and of a hierarchy of secondary spiral windows. We have also found that the shape and the way this hierarchy is constructed depend on the energy of the system. This study is performed through the analysis of intersections of stable and unstable manifolds in the $x-\dot{x}$ Poincaré phase plane.

© 2017 Elsevier Inc. All rights reserved.

1. Introduction

The phenomenon of escapes from a dynamical system, especially the escape of stars from stellar systems has been an active field of research during the last decades [1–8,11,12,16–18,20,21]. In [11], Contopoulos explored escapes from dynamical systems representing the central part of perturbed galaxies, with Hamiltonian [19]

$$\mathcal{H}=\frac{1}{2}(\dot{x}^2+\dot{y}^2)+\frac{1}{2}(x^2+y^2)-\mu x^2y^2 . \quad (1)$$

This system have quadruple symmetry and four openings to infinity. Every opening is bridged by an unstable periodic orbit called a Lyapunov orbit, which governs the escape to infinity from the potential well. The asymptotic curves of the Lyapunov periodic orbits make infinite rotations around some "limiting curves". The proportion of escaping orbits and the direction of escape depend on the topology of the asymptotic surfaces of the Lyapunov periodic orbits. This is investigated in [12] by Contopoulos and Kauffman. The "basins" of escape toward different directions, and of the fast and the slow escapes for various values of the perturbation parameter (μ) are determined. In 1996, Siopis et al. [18] performed a numerical study of the escape properties of three two–dimensional, time–independent potentials possessing different symmetries. It was found, for all three cases, that (i) there is a rather abrupt transition in the behaviour of the late–time probability of escape, when the value of a coupling parameter, μ, exceeds a critical value, μ_2. For $\mu > \mu_2$, it was found that (ii) the escape probability manifests an initial convergence towards a nearly time–independent value, $p_0(\mu)$, which exhibits a simple scaling that may be universal. However, (iii) at later times the escape probability slowly decays to zero as a power–law function of time. Finally, it was found that (iv) in a statistical sense, orbits that escape from the system at late times tend to have short time Lyapunov exponents which are lower than for orbits that escape at early times. Navarro and Henrard [17] examined the

E-mail address: jf.navarro@ua.es

http://dx.doi.org/10.1016/j.amc.2017.01.040
0096-3003/© 2017 Elsevier Inc. All rights reserved.

shape of the windows through which test particles may escape in the simplified Hamiltonian system (1). Here, the authors analyzed the shapes and sizes of the windows of escape of stars from a simplified galactic model, founding that they consist of a "main window" and of a hierarchy of secondary windows. A very large part of the main window is actually made of "just passing through"stars and may not be very interesting for galactic studies. Hence the importance of the secondary windows, their intricate spiral structures and the fractality of the basin boundaries.

In 2004, Contopoulos and Efstathiou [13] studied in detail the form of the asymptotic manifolds of a central unstable periodic orbit. The form of the escape regions and the infinite spirals of the asymptotic manifolds around the escape regions were given, as well as the computation of the escape rate for different values of the energy and the percentage of orbits that escape after a finite number of iterations. The problem of the escape in galactic potentials has been recently revisited by Zotos [20,21], who has investigated the structure of the phase space of the dynamical system described by (1).

In [5], Barrio and coworkers explored the appearance of different kinds of fractal structures in Hénon–Heiles potentials in the unbounded range. In [6], they found that open Hamiltonians present safe bounded regular regions inside the escape region that have significant size and that can be located with precision. Therefore, it is possible to find regions of nonzero measure with stable periodic or quasi–periodic orbits far from the last KAM tori and far from the escape energy. Later, in [7], the authors make use of recent computational techniques in the numerical study of qualitative properties of Hamiltonian systems of two degrees of freedom. These numerical methods are based on the computation of the OFLI2 Chaos Indicator, the Crash Test and exit basins and the skeleton of symmetric periodic orbits.

The aim of this article is to better understand the properties of the escape of orbits in a simple local galactic Hamiltonian describing the motion near the center of a elliptical galaxy. In the present work, we study the potential

$$W(x,y) = \frac{1}{2}(\omega_1^2 x^2 + \omega_2^2 y^2) - \mu\left[\beta(x^4+y^4) + 2\alpha x^2 y^2\right], \tag{2}$$

where ω_1, ω_2 are the unperturbed frequencies of oscillation along the x and y axis respectively, $\mu > 0$ is the perturbation strength, and α and β are positive parameters [9]. Here, we analyze the case $\omega_1, \omega_2 = \omega = 1,$ that is, the 1: 1 resonance case. A description of the Hamiltonian to the potential (2), given by

$$\mathcal{H} = \frac{1}{2}(\dot{x}^2 + \dot{y}^2) + \frac{1}{2}(x^2+y^2) - \mu\left[\beta(x^4+y^4) + 2\alpha x^2 y^2\right], \tag{3}$$

can be found in [10]. It is actually a member of the Verhulst family of galactic potentials (see [19] for a review).

We will compute the asymptotic manifolds to the Lyapunov periodic orbits in the openings of the potential in order to show that the mechanism described in [17] is valid also in this case. As in the simplified model described there, we show that the intricate spiral structures of the secondary windows governs the rate of escape: there are "first order", infinitely winding spirals, but also "second order" spirals composed themselves of an infinity of layers, "third order" spirals formed by an infinity of second order spirals, and so on.

We have found that the shape and size of the spirals, as well as the number of "main" first order spirals, depend on the energy of the system. We show this fact carrying out the analysis for two values of the energy.

In order to investigate the size, shape and properties of the regions of phase space leading to escape, it is necessary to understand the geometry of the stable manifolds to the "guardian" periodic orbits. We will do so by investigating the intersections of the stable and unstable manifolds with a surface of section. This has been the strategy of Contopoulos and coworkers [11,12] and Navarro and Henrard [17].

In our analysis, we will take as surface of section the plane $y = 0,$ and use a sixth order expansion of the stable and unstable manifolds in order to obtain precise initial conditions far enough from the unstable "guardian orbit" to start a safe computation of these manifolds.

2. Symmetries of the problem

There are two different cases of the xy plane in the Hamiltonian (3), depending on the relation between α and β: (a) $\alpha > \beta$. (b) $\beta > \alpha$. In all these situations, there is a critical value of the energy (h_c) such that, for larger values of h, the potential well opens up to infinity and test particles may escape. This critical value is given by

$$h_c = \frac{1}{8\mu(\alpha+\beta)}$$

when $\alpha > \beta$, and

$$h_c = \frac{1}{16\mu\beta}$$

if $\alpha < \beta$. These values can be computed using the method described in [9]. The curves of zero velocity of the Hamiltonian system (3) are given by the relation

$$f(x^2, y^2) = h - \frac{1}{2}(x^2+y^2) + \mu\left[\beta(x^4+y^4) + 2\alpha x^2 y^2\right] = 0\,. \tag{4}$$

Download English Version:

https://daneshyari.com/en/article/5775787

Download Persian Version:

https://daneshyari.com/article/5775787

Daneshyari.com