Applied Mathematics and Computation 309 (2017) 107–132

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Numerical asymptotic stability for the integro-differential equations with the multi-term kernels

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A R T I C L E   I N F O

MSC:
45K05
65D30
34K30
65L07
65M12

Keywords:
The classes of integro-differential equations
Completely monotonic kernel
The second order backward difference type methods
Convolution quadrature
Weighted l^1 asymptotic stability

A B S T R A C T

We prove that a second order backward difference semi-discretization approximation for the integro-differential equations with the multi-term kernels preserves the weighted asymptotic integrabilities of continuous solutions. The method of proof extend and simulate numerically those introduced by Hannsgen and Wheeler, relying on deep frequency domain techniques.

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1. Introduction

The celebrate d integro-differential equations [3–5]

$$u'(t) + \int_0^t L(t-\tau)\,u(\tau)\,d\tau = 0, \qquad t > 0, \tag{1.1}$$

$$u(0) = u_0,$$

are ones of the most paradigmatic examples of thermodynamics, electrodynamics, continuum mechanics, and population biology which have the asymptotic behavior for large time of solutions [3]. In the present, we let

$$L(t) = \sum_{j=1}^{n} a_j(t)\,L_j, \tag{1.2}$$

where the spectral family $\{E_\lambda\}$ corresponding to L is some fixed resolution of the identity, which needs to be common to all the L_j which are densely defined self-adjoint linear operator in a Hilbert space $\mathbf{H}$. The initial data u_0 is prescribed element of $\mathbf{H}$. The functions $a_j(t),\ j = 1, 2, \ldots, n,$ satisfy

$$\int_0^1 a_j(t)\,dt < \infty \quad \text{and} \quad 0 \le a_j(\infty) < a_j(0^+) \le \infty,$$

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http://dx.doi.org/10.1016/j.amc.2017.03.046
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$$a_j(t) \text{ is completely monotonic on } (0,\ \infty), \qquad j = 1, 2, \ldots, n. \tag{1.3}$$

The asymptotic analysis of the problem (1.1) have been studied by various authors [1–6]. When $n = 1$, the asymptotic estimates

$$\int_0^\infty \|u(t)\|\, dt \le C\,\|u_0\|, \tag{1.4}$$

have been established in [1,2], where C is a positive constant independent of $u(t)$, and $\|.\|$ denotes the norm in $\mathbf{H}$, when the kernel $a_1(t)$ satisfies

$$a_1(t) \in C(0,\ \infty) \bigcap L^1(0,\ 1) \text{ is nonconstant, nonnegative, nonincreasing, convex,}$$

$$\text{and } -a_1'(t) \text{ is convex on } (0,\ \infty). \tag{1.5}$$

But, when $n \ge 2$ Noren conjecture that (1.4) holds if each $a_j(t),\ j = 1,\ 2,\ \ldots,\ n$ satisfies (1.5), see [4, p. 390], or [5, p. 550]. It is known that the completely monotonic kernels (1.3) satisfies (1.5). Noren, [4, Theorem 1] gives sufficient conditions such that (1.4) holds when $n \ge 2$.

Hannsgen and Wheeler show in [3, Corollaries 2.2 and 2.3] that, for the completely monotonic kernels $a_j(t),\ j = 1,\ 2,\ \ldots,\ n$ satisfying some reasonable assumptions,

$$\int_0^\infty \rho(t)\,\|u(t)\|\, dt \le C\,\|u_0\|, \tag{1.6}$$

where $\rho(t)$ is a weight function.

The estimate (1.6) was used in [3] to estimate the resolvent kernel

$$U(t) = \int_{\lambda_0}^\infty u(t;\ \lambda_1,\ \lambda_2,\ \ldots,\ \lambda_n)\, dE_\lambda \tag{1.7}$$

of the problem (1.1), where $u(t;\ \lambda_1,\ \lambda_2,\ \ldots,\ \lambda_n) = u(t;\ \lambda_j)$ is the solution of the scalar problem

$$u_t(t;\ \lambda_j) + \int_0^t (\lambda_1 a_1(t-\tau) + \lambda_2 a_2(t-\tau) + \cdots + \lambda_n a_n(t-\tau))\, u(\tau;\ \lambda_j)\, d\tau = 0,\ t > 0, \tag{1.8}$$

$$u(0;\ \lambda_j) = 1, \qquad \lambda_j \ge \lambda_0,\quad j = 1,\ 2,\ \ldots,\ n.$$

From (1.7) we see that (1.6) implies

$$\int_0^\infty \rho(t)\,\|U(t)\|\, dt \le C, \tag{1.9}$$

so that the resolvent formula

$$y(t) = U(t)y_0 + \int_0^t U(t-\tau)f(\tau)d\tau \tag{1.10}$$

can be used to yield information about the asymptotic behavior of the solution $y(t)$ as $t \to \infty$ for the initial value problem

$$y'(t) + \int_0^t L(t-\tau)\, y(\tau)\, d\tau = f(t), \qquad t > 0, \tag{1.11}$$

$$y(0) = y_0,$$

where y_0 and $f(t)$ are prescribed elements of $\mathbf{H}$. The requirement that the $L_j,\ j = 1,\ 2,\ \ldots,\ n$, have spectral decompositions with respect to a common resolution of the identity $\{E_\lambda\}$ greatly restricts the applicability of the result (1.9), but see [3] for some applications, including a linear model for heat flow in a rectangular, orthotropic material with memory in which the axes of orthotropy are parallel to the edges of the rectangle.

The goal of this paper is to introduce a numerical scheme that preserves this weighted $L^1(\rho; 0, \infty; \mathbf{H})$ property at the numerical level. This issue is relevant from a computational point of view since, as has been observed in a number of contexts (fractional wave equation, evolution equation with positive type memory term, sub-diffusion equation, etc. [20–23]), the convergence property in the classical sense of numerical analysis (a property that concerns finite-time horizons) is not sufficient to ensure the asymptotic behavior of the PDE solutions to be captured correctly. The fact that the numerical approximation schemes preserve the decay properties of continuous solutions can be considered as a manifestation of the property of PDE solutions.

We refer to [7,8] for a different attempt in this direction, using the backward Euler method for the time of problem (1.1)–(1.3), are shown to preserve the weighted asymptotic decay properties. Achieving the results better than first-order accuracy in time has proven to be challenging, however in some case extrapolation methods [24,25] offer a means to improve the accuracy of a low-order method.

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