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Analysis of a channel and tube flow induced by cilia

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ABSTRACT

Power law metachronal wave motion, responsible for the cilia transport is investigated in this paper using numerical tools. The dynamical analysis is made in channel and in tube to demonstrate the quantitative effect of the geometry. Similarity transformations are employed to convert the governing partial differential equations into a set of coupled ordinary differential equations. A swift and accurate collocation algorithm is applied to the boundary value problem (BVP) of coupled ordinary differential equations. A nondimensional graphical analysis of the waving amplitude is reported by varying the flow consistency and flow behavior indices.

1. Introduction

The hair-like organisms flagella and cilia utilize the biochemical energy in most of the life functions to transport fluids and propel cells. They play an efficient role while dealing with the non-Newtonian biological fluids. Unfortunately, the cilia transport phenomena did not attract much attention right after its discovery in 1675 by the Dutch light microscopist Antoni Van Leeuwenhoek. However, during the last thirty years an understanding of ciliary structure and function has been approached. The cilia transport has attracted much attention these days. Recently, Vilfan *et al.* [1] reported the successful actuation of the artificial micrometer cilia structures.

These complex organelles have been examined by a number of experimental techniques, particularly Transmission Electron Microscopy (TEM). With this understanding, it is evident that the differentiation between eukaryotic cilia and flagella is not rigorous. Cilia and flagella are organelles that are primarily used for the transportation of the cell. They propel the cell by flicking back and forth. Cilia are short, there are many per cell and they are reminiscent of hairs whereas flagella are longer, there are far fewer per cell and they are reminiscent of a tail. Cilia or flagella shows a variety of beating patterns depending on the surrounding geometry as described in literature [1–6], the purpose of beating (locomotion, ingestion, mixing, etc.) is regulation.

In general, a two-stroke recovery effective motion is displayed by the beating pattern of an individual cilium, the recovery (or backward) stroke is executed by bending the cilium towards itself and the nearest boundary, minimizing by the drag on the surrounding fluid in the backward whereas during the effective stroke, the cilium extends into the fluid, dragging the maximum volume of fluid forward. In some cells, during the recovery stroke the cilium also tilts with respect to the vertical plane described by the effective stroke displaying a three-dimensional pattern [7,8].

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During the combined motion of cilia, they beat in an organized manner such that upper layer of cilia is deformed in a wave-like manner. These waves, known as metachronal waves, are due to a small angular difference between neighboring cilia, and are just like the waves formed by waving spectators in stadiums. This collective motion of cilia has been used in biological studies [9].

According to the relationship between the forward stroke of the motile cilia and their dynamics, different types of metachronal waves are classified. The symplectic beat pattern is formed, when the forward stroke and propagative metachronal wave are unidirectional. In contrast antiplectic beat coordination is formed if both directions are opposite to each other. It is noticed that the antiplectic metachrony pattern is extensively used as compared with the symplectic wave because the prior is classified by the separation of collective cilia during the forward stoke, permitting them to push more fluid capacity and thus mounting the stoke effectiveness [10,11].

For the locomotion of mucus by cilia, an arrangement of shear-thinning and elastic effects is assumed. It can diminish the fluid flow opposition at the cilia layer and preserve a semi rigid plane at the superior mucus layer, which agree to the most favorable transport of particles in the respiratory trail. Because of the significance of mucus, remarkable research has been conducted to describe the tracheobronchial mucus transportation [11–13]. Mucus is a model of a non-Newtonian fluid, so to discuss the rheological fluid motion we consider the power law fluid model, the simplest and most commonly used rheological models to approximate shear-thickening and shear-thinning fluids over a large variety of flow conditions. Therefore, to compute fluid transfer of particles in the respiratory path, the power law model is considered. The term cilia as used in this paper are restricted to ciliated epithelium and will not take in flagella.

The aim of the present paper is to analyze the difference between the channel and tube flow of power law fluid induced by the metachronal wave of ciliary motion. The equation of motion for the channel and tube are solved by the numerical technique 'Generalized Collocation Method' (GCM) after converting the PDE's to ODE's using the similarity transformation. The quantitative analysis is made for the pressure gradient, volume flow rate and velocity profile for both channel and tube geometry. The graphical results for the emerging parameters are presented in the last section.

2. Governing equations

2.1. Channel geometry

Consider a infinite length channel in (x, y) plane with ciliated walls. Let the width of the channel be a and ε be the nondimensional measure associated with the channel width. The wave produced by the ciliary motion propagates in the x-direction, with a velocity c. The cilia motion takes place between the two walls mounted at

$$y = \pm h = \pm a\left[1 + \varepsilon \cos\left(\frac{2\pi}{\lambda}(x - ct)\right)\right] \tag{2.1}$$

To discuss the transport phenomena for a power law fluid in the channel, we will consider the following governing equations

$$\nabla.\mathbf{v} = 0 \tag{2.2}$$

$$\rho\frac{d\mathbf{v}}{dt} = \nabla.\mathbf{T} \tag{2.3}$$

where

$$\mathbf{T} = -p\mathbf{I} + \mathbf{S} \tag{2.4}$$

ρ is the density of fluid, p is the pressure and

$$\mathbf{S} = M^{*}n\mathbf{A}_1 \tag{2.5}$$

provided that

$$M^{*} = \left(\frac{1}{2}tr(\mathbf{A}_1^2)\right)^m \tag{2.6}$$

where n and m are the flow consistency and flow behavior indices and $\mathbf{A}_1$ is the First Rivlin Erickson tensor and is given by

$$\mathbf{A}_1 = \nabla\mathbf{v} + (\nabla\mathbf{v})^T, \tag{2.7}$$

In the component form we can write the velocity as

$$\mathbf{v} = [u(x, y, t), v(x, y, t)]$$

and therefore Eqs. (2.2) and (2.3) can be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.8}$$