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Constructive proof of Lagrange stability and sufficient – Necessary conditions of Lyapunov stability for Yang–Chen chaotic system[☆]

Xiaoxin Liao [a,b], Guopeng Zhou [a,*], Qigui Yang [c], Yuli Fu [d], Guanrong Chen [e]

[a] Institute of Engineering and Technology, Hubei University of Science and Technology, Xianning 437100, PR China
[b] College of Automation, Huazhong University of Science and Technology, Wuhan 430074, PR China
[c] School of Mathematics, South China University of Technology, Guangzhou 510641, PR China
[d] School of Electronics and Information Engineering, South China University of Technology, Guangzhou 510641, PR China
[e] Department of Electronic Engineering, City University of HongKong, Hong Kong Special Administrative Region, PR China

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ABSTRACT

This paper studies the stability problem of Yang–Chen system. By introducing different radial unbounded Lyapunov functions in different regions, global exponential attractive set of Yang–Chen chaotic system is constructed with geometrical and algebraic methods. Then, simple algebraic sufficient and necessary conditions of global exponential stability, global asymptotic stability, and exponential instability of equilibrium are proposed. And the relevant expression of corresponding parameters for local exponential stability, local asymptotic stability, exponential instability of equilibria are obtained. Furthermore, the branch problem of the system is discussed, some branch expressions are given for the parameters of the system.

1. Introduction

Since Lorenz chaotic system was found in 1963 [1,2], chaotic systems have attracted many researchers' interests, and great progress has been made in many areas. Not only basic properties and dynamical behaviors of Lorenz system have been widely studied, but also chaotic control, chaotic synchronization, and other applications have been taken cared of [3–28]. Furthermore, some new chaotic systems which are not equivalent to Lorenz system were found, i.e., Rössler system [4], Chua system [5], Chen system [8,9], Lü–Chen system [10], Yang–Chen system [13], etc. The authors in [31–36] study the chaotic systems by using FPGA method to realize the simulation.

It is generally believed that the conditions for producing chaos of a continuous dynamical system include: (I) the system has at least one positive Lyapunov exponent in a small area; (II) the system is ultimately bounded (or Lagrange stable), namely, the trajectory of the system far away from the equilibrium point converges to a specific bounded set. Furthermore, computing Lyapunov exponent is significant on the condition that (II) holds. Therefore, the proof of ultimate boundedness for dynamical systems is a core and critical problem. Soviet academician Leonov is the first one who studied this problem.

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* Corresponding author.
E-mail addresses: zhgpeng@126.com, zhgpeng@mail.hbust.com.cn (G. Zhou).

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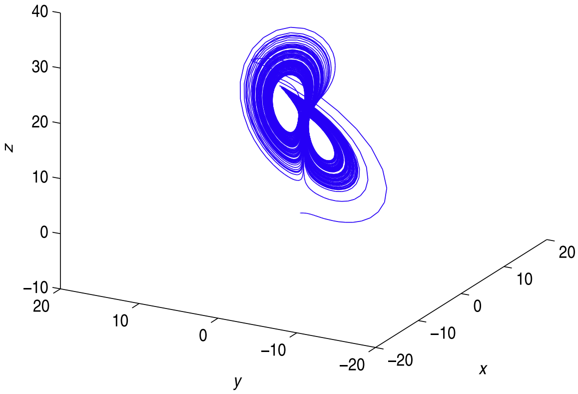

Fig. 1. The phase diagram of Yang–Chen system.

He respectively obtained a cylindrical estimator and one oval estimator for Lorenz system [14–16,25]. Based on the above results, we not only gave a simplified proof for of ultimate boundedness of Lorenz system, but also improved and extended the well-known results [23,24]. Particularly, we first proposed a new concept of global exponential stable in sense of Lagrange [23]. Recently, by using geometric and algebraic methods, we proposed a constructive proof of globally exponentially attractive set for Chen system [18].

Lyapunov stability of equilbria for nonlinear dynamical systems is important. In the literature [25], we have completed Lyapunov stability analysis of equilibrium point of Lorenz system. Some very simple algebraic sufficient and necessary conditions of global exponential stability, global asymptotical stability, and instability were given, and the results were applied to chaotic control very well. But for Yang–Chen system, does it has the same result?

In this paper, we focus on studying Lagrange stability and Lyapunov stability of Yang–Chen system. For convenience of our discussion, Yang–Chen system is described as follows [13]:

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = a(y-x), \\ \dfrac{\mathrm{d}y}{\mathrm{d}t} = dx - xz, \\ \dfrac{\mathrm{d}z}{\mathrm{d}t} = xy - bz, \end{cases} \tag{1}$$

where x, y, $z \in R$ are state variables, a, b, d are parameter constants, and a, b are always assumed to be positive values. Let $X(t) = (x(t), y(t), z(t))$ with $X(t) = X(t, t_0, X_0)$, and X_0 is the initial value of $X(t)$. It has three equilibria $S_0(0, 0, 0), S_{\pm}(\pm\sqrt{bd}, \pm\sqrt{bd}, d)$. When $a = 10, b = 1, d = 16$, system (1) has a chaotic attractor as it is shown in Fig. 1.

Since it has the property of invariance and symmetry (i.e., $(-x, -y, z) \to (x, y, z)$), asymptotic behavior of Yang–Chen system will be studied by transforming three-dimensional state (x, y, z) into two-dimensional plane (y, z). By introducing different radial unbounded Lyapunov functions in different regions, global exponential attractive set of Yang–Chen chaotic system is constructed with geometrical and algebraic methods. Then, simple algebraic sufficient and necessary conditions of global exponential stability, global asymptotic stability, and instability of equilibrium S_0 are proposed. And we also obtain the relevant expression of the corresponding parameters for local exponential stability, local asymptotic stability, exponential instability of equilibria $S_{\pm}$. Furthermore, the branch problem is discussed, some branch expressions are given for the parameters of Yang–Chen system. Lagrange stability and Lyapunov stability conditions presented here are very useful for studying how chaos produced, chaos control, chaos synchronization, and other applications.

The rest of this paper is organized as follows. In the second section, some necessary preparations are given. In Section 3, global exponential attractive set will be presented with a constructive proof. Algebraic sufficient and necessary conditions of Lyapunov stability of equilibrium S_0 are proposed in Section 4. In Section 5, the conditions of local stability of equilibria $S_{\pm}$ are presented. In Section 6, the branch value expressions of different parameters are obtained. Section 7 is the simulation to show the correctness of the results presented in the previous sections. And the final section is the conclusion.

2. Preliminaries

We will discuss the properties of Lagrange stability, Lyapunov stability and bifurcation for system (1). For this purpose, some definitions and lemmas are presented in this section.

Definition 1. If there exists a compact set $\Omega \subset R^3$ for all $X_0 \in \Omega^c \subseteq R^3$ and $\rho(X(t), \Omega) \triangleq \lim_{t\to+\infty} \inf_{Y\in\Omega} \|X(t) - Y\| = 0$ holds, then the compact set Ω is called a global attractive set of system (1). That is, all solutions of system (1) are ultimately bounded. The complementary set Ω^c of Ω is called global attractive domain.