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Family of a-point b-ary subdivision schemes with bell-shaped mask

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ABSTRACT

In this paper, we present a generalized Refine-Smooth algorithm to design a family of a-point b-ary approximating subdivision schemes with bell-shaped mask, where $a \geq 3$ and $b \geq 2$. We use the combination of corner cutting b-ary subdivision scheme and weighted average of $(b+1)$-points to construct the proposed family. We demonstrate that the proposed family has smaller complexity and support width and higher continuity than the existing Refine-Smooth subdivision schemes. We also study the shape preserving properties of the proposed family. In addition, it is observed that the proposed family is suitable for fitting the locally noisy, oscillatory, and irregular data.

1. Introduction

subdivision scheme is an important research subject in computer aided geometric design, computer graphics, geometric modeling and related areas, due to its superior properties, such as numerical stability, low computational complexity and high efficiency. It has many applications in scientific areas. There are many techniques for the construction of subdivision schemes [3,8,13–16]. But mathematical computation of the mask/weights of a subdivision scheme by using Refine-Smooth (RS) technique is easy comparative to the other techniques. Nowadays the RS-algorithm is very popular and simplest algorithm for the construction of binary subdivision schemes. In 1980, Lane and Riesenfeld [9] proposed an algorithm for the construction of binary B-spline schemes, known as Lane–Riesenfeld algorithm. Later on Cashman et al. [4] presented the Lane–Riesenfeld algorithm in the framework of RS-algorithm. According to Cashman et al. [4], the $(l+1)$-degree B-spline scheme can be constructed by using the midpoint averaging interpolatory subdivision scheme

$$\begin{cases} p_{2i}^{k,1} = p_i^k, \\ p_{2i+1}^{k,1} = \frac{1}{2}\left(p_i^k + p_{i+1}^k\right), \end{cases}$$

and average of two adjacent points

$$p_i^{k+1} = \frac{1}{2}\left(p_i^{k,t-1} + p_{i+1}^{k,t-1}\right),$$

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as refining and smoothing operators, respectively. Cashman et al. [4] proposed another family of RS-schemes by using 4-point interpolatory subdivision scheme

$$\begin{cases} p_{2i}^{k,1} = p_i^k, \\ p_{2i+1}^{k,1} = \frac{1}{16}\left(-p_{i-1}^k + 9p_i^k + 9p_{i+1}^k - p_{i+2}^k\right), \end{cases}$$

and weighted average of four adjacent points

$$p_i^{k+1} = \frac{1}{16}\left(-p_{i-1}^{k,t-1} + 9p_i^{k,t-1} + 9p_{i+1}^{k,t-1} - p_{i+2}^{k,t-1}\right),$$

as refining and smoothing operators, respectively. Mustafa et al. [10] constructed a family of RS-schemes. Their refining operator is the 5-point relaxed approximating scheme

$$\begin{cases} p_{2i}^{k,1} = \frac{1}{128}\left(-8p_{i-2}^k + 72p_{i-1}^k + 72p_i^k - 8p_{i+1}^k\right), \\ p_{2i+1}^{k,1} = \frac{1}{128}\left(-3p_{i-2}^k + 12p_{i-1}^k + 110p_i^k + 12p_{i+1}^k - 3p_{i+2}^k\right). \end{cases}$$

They have borrowed the smoothing operator of Cashman et al. [4]. Ashraf et al. [2] proposed a similar family of RS-schemes by using the 6-point interpolatory subdivision scheme

$$\begin{cases} p_{2i}^{k,1} = p_i^k, \\ p_{2i+1}^{k,1} = \frac{1}{256}\left(3p_{i-2}^k - 25p_{i-1}^k + 150p_i^k + 150p_{i+1}^k - 25p_{i+2}^k + 3p_{i+3}^k\right), \end{cases}$$

and weighted average of six adjacent points

$$p_i^{k+1} = \frac{1}{256}\left(3p_{i-2}^{k,t-1} - 25p_{i-1}^{k,t-1} + 150p_i^{k,t-1} + 150p_{i+1}^{k,t-1} - 25p_{i+2}^{k,t-1} + 3p_{i+3}^{k,t-1}\right),$$

as refining and smoothing operators, respectively. Recently Mustafa et al. [11] presented a family of RS-schemes. They have used the 3-point approximating binary scheme

$$\begin{cases} p_{2i}^{k,1} = \frac{1}{32}\left(9p_{i-1}^k + 22p_i^k + p_{i+1}^k\right), \\ p_{2i+1}^{k,1} = \frac{1}{32}\left(p_{i-1}^k + 22p_i^k + 9p_{i+1}^k\right), \end{cases}$$

as the refining operator. They have also used the smoothing operator of Cashman et al. [4]. The following motivation, plays an important role behind the construction of our family of RS-schemes.

1.1. Motivation

The RS-algorithms have been proposed by [2,4,10,11] for the construction of binary subdivision schemes. The construction of higher arity subdivision schemes by RS-algorithm has not been presented by anyone. The higher arity subdivision schemes have some advantages over the lower arity schemes.

- Higher arity subdivision schemes can improve the smoothness of the limit function with small support.
- The convergence rate of higher arity schemes is faster than the convergence rate of lower arity subdivision schemes. Because at each iteration the new sequence of newly inserted control points has b (arity) times as many points as the previous sequence of old control points. In other words, the computational cost decreases by increasing the arity of the scheme.

In addition, the proposed family of a-point b-ary subdivision schemes has some particular advantages over the existing RS-schemes. Some of them are:

- Our family of schemes has the least complexity and support size while it has higher smoothness than the schemes proposed by [2,4,10,11] (See Table 1).
- Our family of schemes eliminates unwanted oscillations and unpleasant artifacts, from the limit curves, caused by the data taken from discontinuous function (See Figs. 5 and 6). In fact, use of the negative weights in the refining operator causes unpleasant artifacts and unwanted oscillations in the limit curve.
- If the initial control points are perturbed with local noise, then by increasing the smoothing stages of the proposed family, noise can be easily removed, whereas the families of RS-schemes [2,4,10,11] can not remove noise by increasing smoothing stages (See Figs. 3 and 4).

The remainder of this paper is organized as follows. Section 2 deals with some basic definitions. In Section 3, the framework for the construction of proposed family of RS- schemes is presented. In Section 4, properties of the family of schemes are analyzed. Applications and comparisons are discussed in Section 5. Conclusions are given in Section 6.