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Stabbing segments with rectilinear objects☆

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ABSTRACT

Given a set S of n line segments in the plane, we say that a region $\mathcal{R} \subseteq \mathbb{R}^2$ is a *stabber* for S if $\mathcal{R}$ contains exactly one endpoint of each segment of S. In this paper we provide optimal or near-optimal algorithms for reporting all combinatorially different stabbers for several shapes of stabbers. Specifically, we consider the case in which the stabber can be described as the intersection of axis-parallel halfplanes (thus the stabbers are halfplanes, strips, quadrants, 3-sided rectangles, or rectangles). The running times are $O(n)$ (for the halfplane case), $O(n \log n)$ (for strips, quadrants, and 3-sided rectangles), and $O(n^2 \log n)$ (for rectangles).

1. Introduction

Stabbing or transversal problems have been widely investigated in computational geometry and related areas. The general idea is to find a region with certain characteristics that intersects (or *stabs*) a collection of geometric objects in a particular way. This family of problems finds applications in multiple areas, such as automatic map generation [28], line simplification [23], regression analysis [3,5], and even in bioinformatics, concretely in simplification of molecule chains for visualization, matching and efficient searching in molecule and protein databases [20].

A substantial amount of research in this area has focused on studying the problem of stabbing a collection of line segments. Several different criteria have been used to define what it means for a region to stab a set of segments: (i) the region must contain exactly one endpoint of each segment, (ii) the region must contain at least one endpoint of each segment, or (iii) the region must intersect all segments (but no restriction on the endpoints is given).

Most previous work on stabbers focuses on criteria (ii) or (iii), but in this paper we deal with criterion (i): we say that a region $\mathcal{R} \subseteq \mathbb{R}^2$ *stabs* a set of segments S if $\mathcal{R}$ contains exactly one endpoint of each segment of S; see Fig. 1(a). Concretely, we study the problem of computing all the combinatorially different stabbers of a given set of segments for some types of stabbing regions. Two stabbing regions are considered *combinatorially different* if and only if the sets of segment endpoints contained in each of them are different, otherwise they are said to be *combinatorially equivalent*. Note that under our definition of stabbing region, the segments can be seen as pair of points (the interior of the segment does not play any role); however, as we will see afterwards, it will be convenient to keep referring to them as segments.

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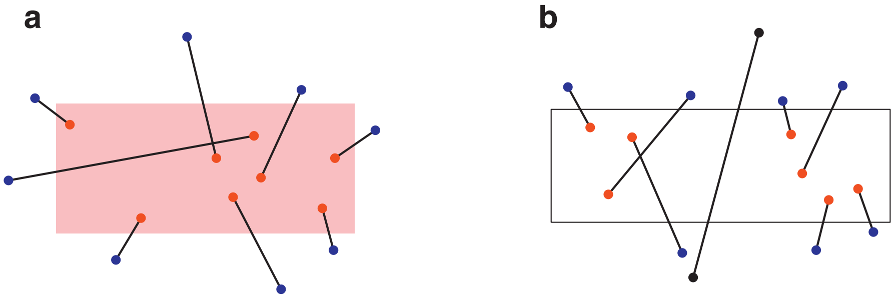

Fig. 1. (a) A set of segments that has a stabbing rectangle. (b) A set of segments for which no stabbing axis-aligned rectangle exists.

Our interest in criterion (i) mainly comes from its differences with respect to the other two criteria. The existence of stabbers in our setting is not always guaranteed, whereas for criteria (ii)-(iii) it is usually easy to find some stabbing region (there are always trivial solutions); this makes our decision question interesting and non-trivial (see Fig. 1). Also, the study of stabbers that require regions to contain exactly one endpoint of each segment fits into the general framework of *classification* or *separability problems* because the stabbers classify endpoints (the ones inside the region and those outside). This is an interesting connection because classification problems arise in many diverse applications, see for instance [26].

1.1. Previous work

We begin discussing previous work on the stabbing model studied in this paper, i.e., criterion (i), and only later we review work on criteria (ii) and (iii).

Perhaps the simplest stabber one can consider is a halfplane, whose boundary is defined by a line. Hence, a stabbing halfplane is defined by its boundary: a line that intersects all segments (note that the complement of a stabbing halfplane is another stabbing halfplane, with the same boundary line). In this context, Edelsbrunner et al. [21] presented an $O(n\log n)$ time algorithm for solving the problem of constructing a representation of all combinatorially different stabbing lines (with any orientation) of a given set of n segments. Moreover, they also gave an $\Omega(n\log n)$ lower bound for the problem. However, the lower bound from [21] does not apply to the decision problem (i.e., determining whether or not there exists a line stabber for a set of segments). Afterwards, Avis et al. [8] gave an $\Omega(n\log n)$ lower bound that holds even for the decision problem in the fixed order algebraic decision tree model.

When no stabbing halfplane exists, it is natural to ask for a stabbing *wedge* (the stabbing region defined by the intersection of two halfplanes). Claverol et al. [14,15] studied the problem of reporting all combinatorially different stabbing wedges of S. The time and space complexities of their algorithm depend on two parameters of S, which in the worst case result in $O(n^3\log n)$ time and $O(n^2)$ space. The authors of [15] also studied some other stabbers such as double-wedges and zigzags. The problem of computing stabbing *circles* of a set S of n line segments in the plane has been studied very recently by Claverol et al. [17], obtaining the following results: (i) a representation of all the combinatorially different stabbing circles for S can be computed in $O(n^2)$ time and space; and (ii) one can report all stabbing circles for a set of parallel segments in $O(n\log^2 n)$ time and $O(n)$ space.

Many of the problems studied for criteria (i) (and (ii)) can be formulated in terms of *color-spanning* objects. In this case, the input is a set of n colored points, with c colors, and the goal is to find an object (rectangle, circle, etc.) that contains *at least* (or *exactly*) k points of each color. Our setting, with criterion (i), is the particular case in which $c = n/2$ and we want to contain *exactly* one point of each color class. Barba et al. [10] considered several problems related to color-spanning objects for criterion (i). In particular, they present algorithms that can compute disks, squares, and axis-aligned rectangles that contain exactly one element of each of the c color classes in $O(n^2c)$ time.

In a very recent paper, Arkin et al. [6] studied the computation of *minimum length separating cycles* for a set of pairs of points. The problem can be seen as that of finding a polygonal stabber of minimum perimeter for a set of line segments. Since the problem is NP-hard, Arkin et al. [6] focused on approximation algorithms for several special cases.

1.1.1. Other stabbing criteria

There is plenty of related work for the two other criteria mentioned before: (ii) containing at least one endpoint, or (iii) intersecting each segment. In the following we briefly mention some of the most relevant papers for these criteria.

Most previous work for criteria (ii) and (iii) has focused on objects in two dimensions. Atallah and Bajaj [7] considered the problem of determining a stabbing line for a collection of objects in the plane. Bhattacharya et al. [11] studied the computation of the shortest stabbing segment of a collection of segments (and lines) with criterion (iii). Arkin et al. [5] considered the problem of computing convex stabbers for sets of segments in the plane, also for criterion (iii). Several types of stabbers have been studied for criterion (ii) in the context of color-spanning objects, namely strips, axis-parallel rectangles [1,18], and circles [2]; all of them can be computed in roughly $O(n^2\log c)$ time, for c the number of different colors. Several papers [12,19,25,30–32,34] have studied problem variants with the goal of optimizing perimeter or area of the convex polygon stabbing a set of segments, using criteria (ii) and (iii).