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Coevolution of discrete, mixed, and continuous strategy systems boosts in the spatial prisoner's dilemma and chicken games

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ABSTRACT

A coevolutionary model by which both the strategy system and strategy value itself are allowed to adapt is established in the framework of spatial 2×2 games. Agents decide to update their behaviors in accordance with a discrete strategy (with a binary strategy set comprising only either cooperation (C) or defection (D)), mixed strategy, or continuous strategy. Because of the evolutionary advantage of the mixed strategy, which allows relatively high cooperators to offer defection to their defective neighbors to avoid exploitation by them, we found that the mixed strategy diffuses to the entire society in most of the dilemma region, and uses robust cooperation to increase the agents' typical payoffs.

1. Introduction

Evolutionary game theory has answered the question of why and how human beings and other animal species evolved altruistic cooperation rather than selfish defection [1–3]. 2×2 (two-player and two-strategy) games such as the prisoner's dilemma (PD), chicken (sometimes synonymously called snowdrift), and stag hunt have provided a solid groundwork for qualitative discussion. Meanwhile, network reciprocity, one of the five fundamental mechanisms for solving social dilemmas by adding "social viscosity," [4] has received intense attention [5–7] because although the central assumption of the model, namely, "playing with neighbors on an underlying network and copying strategies from them," is simple, it still seems a plausible explanation of how cooperation enables survival in any real context. Because of this, there have been many studies on the subject of the spatial PD (SPD) game over the last couple of decades.

One central premise assumed in most previous works is that an agent has a binary strategy of either cooperation (C) or defection (D), which are, hereafter, called discrete strategies. What can be observed in the real world obviously is more complex, where players may offer a halfway point between perfect cooperation and perfect defection in some contexts; in other contexts, players may rely on a stochastic action instead of a deterministically clear offering. In this sense, it is important to study mixed strategies whereby an agent stochastically offers C or D according to their strategy value defined by a real number of either 0 (=D) or 1 (=C) and continuous strategies whereby an agent directly provides a halfway offer based on their real number strategy value. With respect to mixed strategies, what-is-called spatial game setting makes agents offer either C or D several times (consistent with the number of neighbors) derived from a same strategy value (a real number between [0, 1]) in a single time-step. At this occasion, there are twofold interpretations in its modeling; one is that a focal agent once draws an offer and applies that offer to all his neighbors, another is that he respectively

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draws different offers based on a same strategy value and applies those respectively. There are obviously expected statistical differences among those two ways, although the previous studies, despite its richness, have paid none of attention on this specific point, which the present study tries to cast a spotlight. Again by observing real human networks, we notice that some people tend to distinctly make binary-like decisions, but some, even in sticking to C or D, may act in a stochastic manner because human decision-making process may be strongly influenced by stochastic perturbations. Also, we find that some other people dislike the extreme offers of C or D, and thus tend to make a halfway offer as a compromising solution. As such behavior appears to be happening around us, it seems that we should explore the effect of allowing those different strategy systems to co-exist in a single society, which has been beyond the scope of previous studies. The present paper represents a reply to this topic.

Merging the spatial game setting with mixed or continuous strategies has been precisely explored in view of comparisons with the discrete strategy system [8,9]. Previous studies on the subject have reported that the equilibria of continuous and mixed strategy systems significantly differ from that of the discrete strategy system if a spatial game setting is imposed instead of well-mixed situation. But they did not mention what happens if those three strategies co-exist, which implies presuming a coevolutionary framework whereby both strategy systems and strategy values themselves are allowed to adjust for agents in their evolutionary dynamics, although some pioneers have focused on stochastic strategy system with view of coevolutionary frameworks [10–12].

The present paper clarifies the point mentioned above by means of intensive numerical approach.

2. Model setup

First off, let us clarify the terminology used in the following text.

The discrete strategy system composed of C or D alone, is a deterministic strategy system, whereby a game player i offers C ($o_i=1$) when their strategy (s_i) is 1, or D ($o_i=0$) when $s_i=0$. When we come to the mixed strategy system, a player stochastically offers $o_i=\{\text{D, C}\}$ according to their strategy s_i. The continuous strategy presumes that o_i is completely consistent with the player's strategy, s_i. In particular, continuous strategy allows a player to offer a halfway offer between [0,1]. Thus, the continuous strategy system is not stochastic but deterministic despite its strategy value being defined by a real value like the mixed strategy system.

When we assume a mixed strategy system, the following two subordinates in the spatial game setting can also be considered. The first sub-case presumes that a focal agent draws a random number k times based on the same strategy value to determine each offer for their respective k neighbors, which is, hereafter, called the "different offers" (DO) case. Contrasting this, the second sub-case presumes that a focal agent draws a random number just once to fix o_i at the value that is used for all their neighbors. This is, hereafter, called a "consistent offer" (CO) case.

At every time step in the present model, each agent in the network (agent i) plays the games described below with their k immediate neighbors and obtains payoffs from all of the games, implying that the accumulated payoff is evaluated. The underlying topology is a two-dimensional lattice graph (hereafter called a lattice) with degree $k=8$, and the total number of agents is $N=10^4$. After each time step, each agent synchronously updates their strategy.

In the case of either a discrete or mixed strategy system, a player receives a reward (R) for each instance of mutual cooperation (C) in which they partake and a punishment (P) for each mutual defection (D). If one player chooses C and the other chooses D, the latter obtains a temptation payoff (T), and the former is labeled a sucker (S). Without losing mathematical generality, a game space can be defined by presuming that $R=1$, $P=0$, $S=-D_r$ and $T=1+D_g$, where D_g and D_r imply a chicken-type dilemma and a stag hunt-type (SH-type) dilemma, respectively [13,14]. Thus, the payoff matrix can be expressed as $\mathbf{M}=\begin{pmatrix} R & S \\ T & P \end{pmatrix}=\begin{pmatrix} 1 & -D_r \\ 1+D_g & 0 \end{pmatrix}$. We explore not only the PD game ($D_g>0$ and $D_r>0$) but also the chicken game ($D_g>0$ and $D_r<0$) by assuming $0\le D_g\le 1$ and $-1\le D_r\le 1$, because some real social dilemmas, such as environmental problems, might be approximated not by a PD but by a chicken-type dilemma [15]. In the case of $S+T>2R$ belonging to the chicken game class (meaning $D_g+D_r>2$ and $D_g>0$ & $D_r<0$), the cooperation level, P_c is no longer a good index for evaluating social efficiency because ST-reciprocity becomes more meaningful than R-reciprocity in obtaining a higher payoff, π [16,17]. Thus, we observe both P_c and π.

In the case of a continuous strategy system, an agent i is allowed to offer s_i as their offer, o_i. Thus, when the agent i who is offering o_i interacts with agent j offering o_j, agent i obtains a payoff $\pi(o_i, o_j)$ [18];

$$\begin{aligned}\pi(o_i,o_j) &\equiv (S-P)\cdot o_i+(T-P)\cdot o_j \\ &\quad +(P-S-T+R)\cdot o_i\cdot o_j+P \\ &= -D_r\cdot o_i+(1+D_g)\cdot o_j+(-D_g+D_r)\cdot o_i\cdot o_j. \end{aligned} \tag{1}$$

An agent causes co-evolution of both their strategy value, s_i, and what strategy system they apply – whether discrete, mixed (either with CO or DO settings), or continuous – in a deterministic way. In this study, the imitation max (IM) system is applied, by which the focal player, i, imitates both the strategy value and the strategy system by which either they or one of their immediate neighbors has attained the maximum accumulated payoff. If there are multiple neighbors with the maximum payoff, one of them is randomly selected.

Again, at each time-step, all agents synchronously play either a PD or chicken game with all of their neighbors on the lattice, after which strategy values and strategy systems are synchronously updated based on the IM for all agents. We