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Uncertain wave equation with infinite half-boundary

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ABSTRACT

Wave equation is a type of second-order and hyperbolic partial differential equation. It is a commonly used tool to model many kinds of wave propagations such as sound wave, electromagnetic wave, water wave and string vibration propagations. Similarly, uncertain wave equation is a type of uncertain partial equation driven by Liu process, which is widely used to model the wave propagation with uncertain noise such as vibrating string in uncertain environment. The existing literature has studied uncertain wave equation with infinite boundary. Since infinite boundary is a much ideal condition, this paper aims at studying the uncertain wave equation with infinite half-boundary.

1. Introduction

Wave equation is a type of second-order and hyperbolic partial differential equation and a commonly used tool to model many kinds of wave propagations, such as sound wave, electromagnetic wave, water wave and string vibration propagations. Specifically, it can be applied into geological prospecting, oil exploration and some other fields. However, noises always exist in the process of wave propagation, which indicates that classical wave equation is not enough to describe the phenomena with noise. Therefore, Orsingher [16] regarded the noise as random noise and introduced stochastic wave equation driven by Wiener process [18].

Uncertainty theory was introduced by Liu [8] as a counterpart of probability theory to model uncertain phenomena associated with belief degrees. It was a branch of mathematical system based on normality, duality, subadditivity and product axioms. In [8], Liu also presented some fundamental concepts, such as uncertain measure to model belief degrees that are the chances of possible events happening, uncertain variable for describing uncertain quantities, uncertainty distribution for describing uncertain variables, and expected value for ranking uncertain variables. Additionally, Peng and Iwamura [17] provided a sufficient and necessary condition for a function being the uncertainty distribution. On the basis of uncertainty theory, some scholars have done lots of work, such as uncertain programming [10], uncertain risk and reliablity analysis [5,13], and uncertain graphs [6,7].

Uncertain process was proposed by Liu [9] to deal with the evolution of uncertain phenomena. Liu process [11] was designed as an important type of uncertain process, which is a Lipschitz continuous uncertain process with stationary and independent increments. Additionally, Liu process plays a significant role in uncertain differential equation which was established by Liu [9] as a special type of differential equation. The existence and uniqueness of solutions of uncertain differential equations were discussed by Chen and Liu [1]. Yao and Chen [20] designed Euler numerical method and Gao [2] designed Milne numerical method for solving uncertain differential equations, respectively. Zhu [21] introduced the uncertain differential equation to optimal control, and Liu [11] applied the uncertain differential equation into finance.

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Uncertain partial differential equation was designed by Yang and Yao [19] to deal with some complex phenomena with uncertain noises. Following that, Gao and Ralescu [4] applied the uncertain differential equation into wave propagations and established uncertain wave equation. Since the velocity of wave propagation is finite, there will exist a paradox if we insist on using Wiener process to model inexact noises, which was indicated in detail in [4]. While, the solution obtained in [4] is under a much ideal condition that the string has infinite length, that is x is defined on the whole real number space. This paper aims at obtaining the solution of uncertain wave equation with infinite half-boundary. The rest paper is organized as follows. Section 2 will be employed to review some basic concepts and properties concerning uncertain variables and uncertain partial differential equations. The solution of an uncertain wave equation with infinite half-boundary will be studied in Section 3 including homogeneous and nonhomogeneous cases. For describing the solution, we will provide the inverse uncertainty distribution of the solutions in Section 4. Finally, we will make a brief summary in Section 5.

2. Preliminaries

This section is devoted to retrospecting some fundamental concepts and properties with respect to uncertain variables and uncertain partial differential equations.

2.1. Uncertainty variable

Let Γ be a nonempty set, and $\mathcal{L}$ a σ-algebra over Γ. Each element Λ in $\mathcal{L}$ is called an event and assigned a number $\mathcal{M}\{\Lambda\}$ to indicate the belief degree that we believe Λ will happen. In order to deal with belief degrees rationally, Liu [8] suggested the following three axioms:

Axiom 1. (Normality axiom) $\mathcal{M}\{\Gamma\}=1$ for the universal set Γ;

Axiom 2. (Duality axiom) $\mathcal{M}\{\Lambda\}+\mathcal{M}\{\Lambda^c\}=1$ for any event Λ;

Axiom 3. (Subadditivity axiom) For every countable sequence of events $\Lambda_1, \Lambda_2, \ldots,$ we have

$$\mathcal{M}\left\{\bigcup_{i=1}^{\infty}\Lambda_i\right\} \le \sum_{i=1}^{\infty}\mathcal{M}\{\Lambda_i\}.$$

Definition 1 (Liu [8])**.** The set function $\mathcal{M}$ is called an uncertain measure if it satisfies the normality, duality, and subadditivity axioms.

The triplet $(\Gamma, \mathcal{L}, \mathcal{M})$ is called an uncertainty space. Furthermore, the product uncertain measure on the product σ-algebra $\mathcal{L}$ is defined by Liu [11] as follows:

Axiom 4. (Product axiom) Let $(\Gamma_k, \mathcal{L}_k, \mathcal{M}_k)$ be uncertainty spaces for $k=1,2,\ldots$. The product uncertain measure $\mathcal{M}$ is an uncertain measure satisfying

$$\mathcal{M}\left\{\prod_{k=1}^{\infty}\Lambda_k\right\} = \bigwedge_{k=1}^{\infty}\mathcal{M}_k\{\Lambda_k\}$$

where Λ_k are arbitrary events chosen from $\mathcal{L}_k$ for $k=1,2,\ldots,$ respectively.

Definition 2 (Liu [8])**.** An uncertain variable is a measurable function ξ from an uncertainty space $(\Gamma, \mathcal{L}, \mathcal{M})$ to the set of real numbers, i.e., for any Borel set B of real numbers, the set

$$\{\xi \in B\} = \{\gamma \in \Gamma \,|\, \xi(\gamma) \in B\}$$

is an event.

Definition 3 (Liu [11])**.** The uncertain variables $\xi_1, \xi_2, \ldots, \xi_n$ are said to be independent if

$$\mathcal{M}\left\{\bigcap_{i=1}^{n}(\xi_i \in B_i)\right\} = \bigwedge_{i=1}^{n}\mathcal{M}\{\xi_i \in B_i\}$$

for any Borel sets $B_1, B_2, \ldots, B_n$.

For describing an uncertain variable, Liu defined uncertainty distribution as follows.

Definition 4 (Liu [8])**.** Suppose ξ is an uncertain variable. Then the uncertainty distribution of ξ is defined by

$$\Phi(x) = \mathcal{M}\{\xi \le x\}$$

for any real number x.

Theorem 1 (Peng–Iwamura theorem [17])**.** *A function $\Phi(x)$: $\Re \to [0, 1]$ is the uncertainty distribution of an uncertain variable if and only if it is a monotone increasing function except $\Phi(x) \equiv 0$ and $\Phi(x) \equiv 1$.*