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Numerical solutions of coupled Klein–Gordon–Zakharov equations by quintic B-spline differential quadrature method

Roshan Thoudam

Department of Mathematics, Sikkim University, 6th Mile, Samdur, Tadong, Gangtok 737102, Sikkim, India

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ABSTRACT

Numerical solutions of the coupled Klein–Gordon–Zakharov equations are obtained by using quintic B-spline based differential quadrature method. A Runge–Kutta fourth method is used for time integration. Stability of the scheme is studied using matrix stability analysis. The accuracy and efficiency of the presented method is shown by conducting some numerical experiments on test problems which includes the motion of single soliton and interactions of two solitons. The numerical results are found in good agreement with the exact solutions.

1. Introduction

The coupled Klein–Gordon–Zakharov (KGZ) equations were introduced by Dendy [1], to model the interactions between the Langmuir waves and the ion acoustic waves in plasma. Denoting the fast time scale of electric field of the electron by the complex function $U(x, t)$ and the deviation of density from its equilibrium by the real function $N(x, t)$, the one-dimensional KGZ system takes the form:

$$U_{tt} - U_{xx} + U + NU + |U|^2 U = 0, \tag{1}$$

$$N_{tt} - N_{xx} = \left(|U|^2\right)_{xx}, \tag{2}$$

with the initial conditions

$$\left.\begin{array}{l} U(x,0) = U_0(x);\ \ U_t(x,0) = U_1(x), \\ N(x,0) = N_0(x);\ \ N_t(x,0) = N_1(x), \end{array}\right\} \tag{3}$$

and the boundary conditions

$$\left.\begin{array}{l} U(a,t) = U_a(t);\ \ U(b,t) = U_b(t), \\ N(a,t) = N_a(t);\ \ N(b,t) = N_b(t), \end{array}\right\} \tag{4}$$

where $x \in \Omega = [a, b] \subset R$ and $0 \le t \le T$.

The initial-boundary value problem (1)–(4) possesses the following conservative quantity:

$$E = \int_a^b \left[|U_t|^2 + |U_x|^2 + |U|^2 + N|U|^2 + \frac{1}{2}|V|^2 + \frac{1}{2}|N|^2 + \frac{1}{2}|U|^4 \right] dx = const, \tag{5}$$

where the potential V is define as $V = -f_x,\ f_{xx} = N_t$.

E-mail address: roshandmc@gmail.com

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Well posedness of the KGZ equations has been proved by Ozawa et al. [2] in three space dimensions and the global solutions of the equations has also been obtained by Boling and Guangwei [3–5]. The stability behavior of solitary waves for KGZ equations are discuss in [6,7]. As coupled equations, KGZ equations have a similar shaped to Zakharov and Klein–Gordon–Schrodinger equations. Analytical methods are applied and exact solutions are obtained in [8–10] for KGZ equations. Li [10] obtained the exact explicit travelling wave solutions for $(n+1)$-dimensional KGZ equations. In [8,9] solitons and conoidal waves of the KGZ equations are obtained by applying the solitary wave Ansatz method, the travelling wave hypothesis method, the (G'/G) method and the mapping method. Solitons are solitary waves with an elastic property. They appear as a result of a balance between weak nonlinearity and dispersion. Solitons retain their shapes and speed after colliding with each other. An extensive overview of the soliton solutions of some well-known partial differential equations have been found in [11–20].

A few numerical methods have been proposed for KGZ equations. Multisymplectic numerical methods that preserve the discrete multisymplectic conservative law had studied by Wang [21] for solitary wave propagation and interaction of the KGZ equations. In [22] Wang et al. derived an explicit and an implicit conservative difference schemes for the KGZ equations. A finite difference scheme with a parameter θ has been proposed by Chen and Zhang [23]. Also, Ghoreishi et al. [24] used Chebyshev Cardinal Functions and employing the operational matrices for derivatives, they reduce the PDE to nonlinear algebraic equations. In [25] Dehghan and Nikpour proposed, the differential quadrature and globally radial basis function methods for the numerical solutions of the KGZ equations.

Differential quadrature method (DQM) was first introduced by Bellman et al. [26] in 1972 to solve ordinary and partial differential equations. This method approximates the derivatives of a function at a certain point using weighted sum of the functional values at certain discrete points. Many authors have used various basis functions to develop various types of DQMs. Legendre polynomials, Lagrange interpolation polynomials, spline functions, radial basis functions, etc. are some of them [26–30] that can be counted. One of the best methods was developed by Shu and Rechards using Lagrange interpolation polynomials as test function [31]. They obtained explicit formulations to compute the weighting coefficients. In the recent years, the DQM has been widely popular due to its easy applicability, stability, high accuracy and adaptation with other numerical methods and many engineering/physics problems have been solved successfully using various DQMs [25,32–44].

In this paper, we present a quintic B-spline differential quadrature method (QBS-DQM) to solve the Klein–Gordon–Zakharov equations. We use quintic B-spline basis functions to compute the weighting coefficients. First the KGZ equations are converted into six partial differential equations (PDEs) and then equations are discretized spatially by QBS-DQM. Then we obtained systems of ordinary differential equations (ODEs) in time. The obtained systems of ODEs are solved using RK4 [45] scheme and consequently the approximate solution is obtained. The numerical solutions of a variety of coupled nonlinear PDEs can be obtained by the application of the proposed method without linearization of the original equation and this method can achieved accurate result with less computational time and less number of grid points.

2. Quintic B-spline differential quadrature method (QBS-DQM)

Let us consider the grid distribution $a=x_0 < x_1 < \cdots < x_N=b$ of the finite interval $[a, b]$. Then, based on the differential quadrature theory, the derivative value $u^{(n)}(x)$ with respect to x at a point x_i is given by

$$u^{(n)}(x_i) = \sum_{j=0}^{N} w_{i,j}^{(n)} u(x_j), \ i = 0, \ 1, \ldots, \ N \text{ and } n = 1, \ 2, \ \ldots, \ N-1 \tag{6}$$

where $w_{i,j}^{(n)}$ represents the weighting coefficients, and N is the number of subintervals in which the solution domain $[a, b]$ is divided.

Let $Q_i(x)$ be the quintic B-splines with knots at the point x_i where the uniformly distributed grid points are chosen as $a=x_0 < x_1 < \cdots < x_N=b$ on the ordinary real axis with $h=x_i-x_{i-1}$ $i=1\ldots, N$. The quintic B-spline $Q_i(x)$ at the knots is given by

$$Q_i(x) = \frac{1}{h^5} \begin{cases} (x-x_{i-3})^5 & x \in [x_{i-3}, x_{i-2}) \\ (x-x_{i-3})^5 - 6(x-x_{i-2})^5 & x \in [x_{i-2}, x_{i-1}) \\ (x-x_{i-3})^5 - 6(x-x_{i-2})^5 + 15(x-x_{i-1})^5 & x \in [x_{i-1}, x_i) \\ (x_{i+3}-x)^5 - 6(x_{i+2}-x)^5 + 15(x_{i+1}-x)^5 & x \in [x_i, x_{i+1}) \\ (x_{i+3}-x)^5 - 6(x_{i+2}-x)^5 & x \in [x_{i+1}, x_{i+2}) \\ (x_{i+3}-x)^5 & x \in [x_{i+2}, x_{i+3}) \\ 0, & otherwise \end{cases}$$

The quintic B-splines $\{Q_{-2}, Q_{-1}, Q_0, Q_1, \ldots, Q_{N-1}, Q_N, Q_{N+1}, Q_{N+2}\}$ form a basis for functions defined over the domain $[a, b]$. Each quintic B-spline covers six elements so that each element is covered by six quintic B-splines. The values of $Q_i(x)$ and its derivative may be tabulated as in Table 1. Using the quintic B-splines as test functions in the fundamental DQM, Eq.