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On stabilizability of switched positive linear systems under state-dependent switching☆

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ABSTRACT

This paper addresses the stabilization of switched positive linear systems by state-dependent switching. We show that if there is a Hurwitz convex (or linear) combination of the coefficient matrices, then the switched positive linear system can be exponentially stabilized by means of a single linear co-positive Lyapunov function. If there is not a stable combination of system matrices, it is shown that the exponential stabilizability is equivalent to a completeness condition on the coefficient matrices. When the switched positive systems can not be stabilized by the single Lyapunov function, we provide a unified criterion for piecewise exponential stabilizability in terms of multiple linear co-positive Lyapunov functions.

1. Introduction

As a special class of hybrid dynamical systems, switched systems have numerous applications in the control of manufacturing systems [1], traffic control [2], automotive engine control and air craft control [3], and many other fields [4]. For a discussion of various issues related to switched systems, see the survey article [4]. The typical switched system is composed of a family of continuous-time or discrete-time subsystems and a rule orchestrating the switch among them. In this paper, we consider the switched linear system:

$$\dot{x} = A_{\sigma(t,x)}x, \tag{1}$$

where x is the state vector in the real vector space $\mathbb{R}^n$. $\sigma : [0, \infty) \times \mathbb{R}^n \to \underline{m} := \{1, 2, \dots, m\}$ is the so-called switching signal. The system matrices $A_i (i \in \underline{m})$ belong to the real $n \times n$ space $\mathbb{R}^{n\times n}$. In general, switching events can be classified, according to the switching type, into time-dependent (depending on the time t only, i.e., $\sigma = \sigma(t)$) and state-dependent (depending on the state x, i.e., $\sigma = \sigma(x)$). For a linear time-invariant (LTI) system $\Sigma_A : \dot{x} = Ax$, if the non-negativeness of initial condition implies that the state x is non-negative at the every $t \geq 0$, then it is called positive system [5]. The switched system (1) is called a switched positive linear system (SPLS) if all its subsystems are positive systems. Recently, the importance of linear switched positive systems has been highlighted by many researchers because of their broad application in communication systems [6], formation flying [7], mathematical networked epidemiology [8,9], and other areas.

The stability and stabilizability of switched positive systems, especially SPLSs, have drawn a lot of attentions in the last decade. One is the stability analysis of SPLSs under arbitrary switching (see, e.g., [10–12]); The other question is whether

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the SPLS is stable or stabilizable when there are some restrictions on the switching signals. These restrictions may be either time-dependent or state-dependent. The stability of SPLSs under time-dependent switching captured wide attention and has more abundant achievements (see, e.g., [13–16] and some references therein). On the other hand, state-dependent stabilizability of SPLSs is a topic only partially explored [17–21]. In [17], the authors discuss the existence of a Hurwitz convex combination of the sysytem (Metzler) matrices. In [17,18], the authors show that some special SPLSs, such as second order systems, two mode systems with rank one difference, can be exponentially stabilized. Similarly, for discrete-time SPLSs, the existence of a Schur convex combination of the system matrices implies state-dependent stabilizability [19]. The necessity is only for some special cases (for example, the second order case [19], cyclic monomial matrix and circulant matrix cases [20]). It should be emphasized that all these results are essentially based on some special restrictions, such as the low dimension, the existence of a Hurwitz convex combination, etc., thus leading to deeper insights into the state-dependent stabilization problem of SPLSs must be endowed with.

This paper will focus on the stabilization issue of SPLSs under state-dependent switching. We aim at exploiting some constructive switching strategies which are applicable to several broad classes of SPLSs. The layout of the paper is as follows. Section 2 states some preliminary results. In Section 3, we provide some stabilization strategies via the single Lyapunov function methods. We first show that the existence of a Hurwitz convex combination of the coefficient matrices implies state-dependent stabilizability, and that the converse is true only for the two mode systems. When the assumption of a Hurwitz combination is not possible, it is shown that exponential stabilizability is equivalent to a completeness condition on the coefficient matrices. Section 4 focuses on piecewise stabilizability. When the stabilization can not be carried out with the help of a single Lyapunov function, we prove exponential stabilizability by using multiple Lyapunov functions. It is worth noting that the proposed results are unified criteria for SPLSs which are without any a priori restriction, in contrast to others in the literature. Finally, Section 5 concludes this paper.

2. Preliminaries

Throughout, for matrix A, B or vector x, y, $A \succeq B(A \succ B)$ or $x \succeq y(x \succ y)$ means that all elements of matrix $A-B$ or vector $x-y$ are non-negative (positive). Similarly, $A \preceq B(A \prec B)$ or $x \preceq y(x \prec y)$ means that all elements of matrix $A-B$ or vector $x-y$ are non-positive (negative). A^T represents the transpose of matrix A. I denotes the identity matrix. $x \neq 0$ means that there exists at least one non-zero entry in vector x. $\|x\| = \sum_{k=1}^{n} |x_k|$, where x_k is the kth element of $x \in \mathbb{R}^n$. $\vec{\mathbf{1}}$ is the vector of all ones.

Before proceeding, we recall some facts which are relevant for this paper.

The following S-procedure for linear version is presented in [22,23].

Theorem 2.1 [22,23]**.** *("S-procedure for linear forms") Let $\sigma_k(y) = y^T s_k + r_k$, where vectors $s_k, y \in \mathbb{R}^n$ and $r_k \in \mathbb{R}$, $k = 0, 1, \ldots, N$. If $\sigma_k(y)$ is regular (i.e., there is one y^* such that $\sigma_k(y^*) > 0$ for $k = 1, \ldots, N$), then the following statements are equivalent for any finite number of constraints N:*

(i) *$\sigma_0(y) \geq 0$ for $y \in \mathbb{R}^n$ whenever $\sigma_k(y) \geq 0$, $k = 1, \ldots, N$.*
(ii) *There exists constants $\tau_k \geq 0$ $(k = 1, \ldots, N)$ such that $\sigma_0(y) - \sum_{k=1}^{N} \tau_k \sigma_k(y) \geq 0$.*

The following lemma which is straightforward from Theorem 2.1 will play a key role in deriving the results of this paper.

Lemma 2.1. *Let vectors w_0, and w_1 be in $\mathbb{R}^n$. If there exists a constant $\tau \geq 0$ such that $w_0 - \tau w_1 \succeq 0$, then for every $x \succeq 0$ in $\mathbb{R}^n$, $x^T w_0 \geq 0$ whenever $x^T w_1 \geq 0$.*

A matrix is a Metzler matrix if its off-diagonal entries are non-negative. A classic result is that an LTI system Σ_A is positive if and only if its system matrix A is a Metzler matrix [5]. A matrix is a Hurwitz matrix if and only if all its eigenvalues lie in the open left half of the complex plane. There are a number of equivalent conditions for a Metzler–Hurwitz matrix and the stability of positive LTI system Σ_A. Some conditions which are relevant for the work of this paper are collected here.

Theorem 2.2 [10,24]**.** *Let $A \in \mathbb{R}^{n\times n}$ be a Metzler matrix, the following statements are equivalent.*

(i) *The LTI system Σ_A is asymptotically stable, i.e., $x \to 0$ as $t \to \infty$.*
(ii) *$A \in \mathbb{R}^{n\times n}$ is a Hurwitz matrix.*
(iii) *There exists a vector $v \succ 0$ in $\mathbb{R}^n$ such that $A^T v \prec 0$.*

Related to statement (iii), we recall a common definition which is a powerful research tool for positive systems. The function $V(x) = x^T v$ is call a linear co-positive Lyapunov function (LCLF) [10] of the positive LTI system Σ_A if $V(x) > 0$ and $\dot{V}(x) = x^T A^T v < 0$ for all non-zero $x \succeq 0$.

Definition 2.1. The SPLS (1) is exponentially stable if there exist constants $\gamma > 0$ and $\delta > 0$ such that the system solution x satisfies $\|x\| \leq \gamma e^{-\delta(t-t_0)} \|x_0\|$ for the initial $x_0 \succeq 0$ and $t \geq t_0$. In addition, The SPLS (1) is called exponentially stabilizable if there exist switching signals σ such that the SPLS (1) is exponentially stable.

In this paper, we will design some effective state-dependent switching strategies $\sigma(x)$ to exponentially stabilize SPLS (1). Notice that if at least one of the individual is asymptotically stable, or equivalently, there exists at least one system matrix