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A simple algorithm for exact solutions of systems of linear and nonlinear integro-differential equations

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ABSTRACT

A Simple algorithm is used to achieve exact solutions of systems of linear and nonlinear integro- differential equations arising in many scientific and engineering applications. The algorithm does not need to find the Adomain Polynomials to overcome the nonlinear terms in Adomain Decomposition Method (ADM). It does not need to create a homotopy with an embedding parameter as in Homotopy Perturbation Method (HPM) and Optimal Homotopy Asymptotic Method (OHAM). Unlike VIM, it does not need to find Lagrange Multiplier. In this manuscript no restrictive assumptions are taken for nonlinear terms. The applied algorithm consists of a single series in which the unknown constants are determined by the simple means described in the manuscript. The outcomes gained by this algorithm are in excellent concurrence with the exact solution and hence proved that this algorithm is effective and easy. Four systems of linear and nonlinear integro-differential equations are solved to prove the above claims and the outcomes are compared with the exact solutions as well as with the outcomes gained by already existing methods.

1. Introduction

Systems of integro-differential equations arise in mathematical modeling of many phenomena, such as activity of interacting inhibitory and excitatory neurons [1], nano-hydrodynamics [2], wind ripple in the desert [3], drop wise condensation [4]. The problems in science and engineering physical phenomena such as currents in electric fields and magnetic fields, pulses in biological chains, industrial mathematics, oscillation theory and control theory of financial mathematics, fluid dynamics mass and heat are modeled by integro-differential equations [5,6]. Presently much attention is paid by the researchers to solve such problems. Different methods are applied to solve such problems numerically as well as analytically [7–13]. Various polynomials like; Taylor, Chebyshev, Daftardar-Jaffari and Adomain's are used to solve systems of linear and nonlinear integro-differential equations [14–18]. Different matrices like; Bernstein operational matrix of derivatives, operational matrix with Block-Pulse functions are applied to solve the same equations [19,20]. The application of the above techniques can give good results but at the same time it creates various difficulties; like, constructing a homotopy in HPM, OHAM and the solution of the corresponding algebraic equations. Calculation of Adomian polynomials to overcome the nonlinear terms in ADM and calculate Lagrange multiplier in VIM, respectively. In 2008, Tahmasbi and Fard [14] have used a new and simple method called the Power Series Method (PSM) to solve Volterra integral equations. Wazwaz et al. used

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VIM and ADM to solve the Volterra integral form of the Lane–Emden equations with initial values and boundary conditions [12,17]. Ebrahimi and Rashidinia used Collocation Method for Volterra–Fredholm Integral and Integro-Differential Equations [21]. Armand el al. give Numerical solution of the system of Volterra integral equations of the first kind [22]. Hesameddint and Asadolahifard [23] solve Systems of Linear Volterra Integro-Differential Equations by Using Sinc-Collocation Method. Biazar et al. used a Strong Method for solving systems of Integro-Differential Equations. Aguirre-Hernández et al. have used polynomial and topological methods to study systems of differential equations as mentioned in [24–26]. In the present paper we apply a series algorithm to solve systems of linear and nonlinear integro-differential equations. The algorithm consist of few steps explained in the coming section which converges easily to the exact solution. The applied algorithm is simple in learning and easy to apply. The paper is divided into five parts. First and second parts consist of introduction related with the manuscript and applied algorithm. Third part is devoted to the application of the algorithm to four systems of linear and nonlinear integro-differential equations. In the fourth part the results and errors are explained in tabular form and finally, conclusions are given in the fifth part. We have used math type and mathematica 7.0 for calculations and numerical simulations.

2. Introduction of the applied algorithm

Consider a system of Integral equations of the form

$$\mu_i(r) = f_i(r) + \int_0^r \kappa_i(r,s)\phi_i(\mu_1(s), \mu_2(s), \mu_3(s), \dots, \mu_m(s))ds, \tag{1}$$

where $\kappa_i(r,s), i = 1,2,3,\dots,m$ are kernels of integral equations and $\mu_i(r), i = 1,2,3,\dots,m = [\mu_1, \mu_2, \dots, \mu_m]^T$ are unknown solutions to be calculated $f_i(r)$ are real valued functions and $\phi_i(\mu(s))$ are linear or nonlinear function of $\mu_1(s), \mu_2(s), \mu_3(s), \dots, \mu_m(s)$.

Let the solution of (1) be given as:

$$\mu_i(r) = \sum_{j=0}^{m} \alpha_{ij} r^j, \tag{2}$$

with initial conditions $\alpha_{i0} = \mu_i(0), i = 1,2,\dots,m$ and also $\alpha_j = \alpha_{ij}, j = 0,1,2,3,\dots,m$.

The coefficients in the above can be determined step by step as: Suppose the solution of (1) be

$$\mu_i(r) = \alpha_0 + \alpha_1 r. \tag{3}$$

where α_1 is unknown constant which can be determined by putting (3) in (1) and with simple calculations we obtained

$$(M_1\alpha_1 - q_1)r + \Phi_1(r^2) = 0, \tag{4}$$

where M_1 is $m \times m$ constant matrix, q_1 is $m \times 1$ constant vector, $\Phi_1(r^2) = [p_{i1}(r^2)], i = 1,2,\dots,m$ and $p_{i1}(r^2)$ are polynomials of order greater than one. Now neglecting $\Phi_1(r^2)$ and comparing the coefficients of r on both sides of (4) we obtained:

$$M_1\alpha_1 - q_1 = 0, \tag{5}$$

from (5) we can find the value of α_1 which is the 1st step. In the next step let the solution of (1) be given as:

$$\mu_i(r) = \alpha_0 + \alpha_1 r + \alpha_2 r^2, \tag{6}$$

where α_0 and α_1 are known and α_2 is unknown constant. Now putting (6) in (1) and solve we get

$$(M_2\alpha_2 - q_2)r^2 + \Phi_2(r^3) = 0, \tag{7}$$

where $\Phi_2(r^3) = [p_{i2}(r^3)]$ is a system of polynomials of order greater than two. Now comparing the coefficients of r^2 and neglecting $\Phi_2(r^3)$ in (7) we obtain

$$M_2\alpha_2 - q_2 = 0. \tag{8}$$

The unknown value α_2 in (8) can be determined easily. If we continue the same procedure for m iterations then we get a series of the following form

$$\mu_i(r) = \alpha_0 + \alpha_1 r + \alpha_2 r^2 + \alpha_3 r^3 + \dots + \alpha_m r^m, \tag{9}$$

which gives an approximate solution for the exact solution of (1) in the given interval.

2.1. Theorem

If $r = r_0$ is an ordinary point of an integro-differential equation then we can always find two linearly independent power series solutions centered at r_0: $\mu(r) = \sum_{j=0}^{m} \alpha_j (r - r_0)^j$. A series solution converges on at least some interval $|r - r_0| < R$, where R is the distance from r_0 to the closest singular point and is called the radius of convergence.