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Phase synchronization between two neurons induced by coupling of electromagnetic field

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ABSTRACT

Based on an improved neuron model with electromagnetic induction being considered, the phase synchronization approaching is investigated on a four-variable Hindmarsh–Rose neuron model by describing the electromagnetic induction with magnetic flux. The effect of time-varying electromagnetic field is described by magnetic flux and the coupling of electromagnetic field is also described by exchange of magnetic flux. It is found that magnetic flux coupling between neurons can induce perfect phase synchronization, the Lyapunov exponent spectrum and local Lyapunov dimension are calculated by using wolf scheme to detect phase synchronization of chaotic time series for membrane potentials. These results confirmed that neurons exposed to external electromagnetic field can induce phase synchronization and appropriate behaviors can be selected. It could give new mechanism explanation for phase synchronization by applying field coupling between neurons.

1. Introduction

Synchronization and anti-synchronization between nonlinear dynamical systems are interesting phenomena, and thus consensus behaviors can be detected. It began from motion of reversed pendulums carried out by Huygens in 1673 [1], phase synchronization or rhythm synchronization has been investigated extensively in different systems. Another interesting example in ecology is that many glowworm can assemble together and shine synchronously [2]. Similar phenomena can be observed in chemical waves [3]. Furthermore, synchronization between chaotic systems began to dram much attention for many researchers since the breakthrough in chaotic circuit finished by Pecora and Carroll in 1990, and the concept of chaos synchronization is defined [4]. Synchronization between chaotic systems mainly discussed complete synchronization, generalized synchronization, phase synchronization, lag synchronization. The investigation for synchronization for oscillators, neuronal activities could be of importance for understand some complex phenomena and also could be helpful for potential application in secure communication, parameter estimation in dynamical systems. As a result, and many effective schemes are proposed to realize synchronization and control of chaos, hyperchaos, and spatiotemporal chaos [5-17]. For example, synchronization transition [10] of chaotic behaviors can be induced by adjusting bifurcation parameter and coupling inten-

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sity. Ref [14]. discussed the mixed synchronization and parameter estimation in chaotic Hindmarsh–Rose neuron model. Ref. [17] suggested nonlinear analysis can be used to predict the collapse of neuronal network. For a brief review, readers can find survey in Refs. [18,19] and references therein. For identical dynamical systems, complete synchronization can be enhanced and developed from phase synchronization, which requires rhythm consensus while amplitude diversity occurs, by increasing the coupling intensity. In fact, realistic dynamical systems could be different in structure and parameter setting, thus phase synchronization could be available than complete synchronization.

In neuronal system, electrical activities can show multiple modes such as quiescent, spiking, bursting and even chaotic behaviors by applying different forcing currents on the neuron. And interspike interval (ISI) is often calculated to detect the mode transition of electrical activities. For example, *Riehle* et al., [20] investigated the rhythm synchronization between different neurons and the existence of synchronization region in brain has been confirmed. Indeed, phase synchronization in different regions of brain is associated with complex biological function. Furthermore, *Montgomery* [21] et al. observed the synchronization of gamma wave. More researchers used to investigate the dynamical properties within the well known neuron models [22–28], while some researchers prefer to setting new reliable neuron models to be consistent with biological experiments [29–32]. For example, *Gu* et al. [30] proposed a new neuron model to detect complex dynamical behaviors in electrical activities in larger parameter region. *Song* et al. [31] investigate the biological function of autapse connection to neurons, further comments can be found in the review [19]. For realistic neuronal systems composed of a large number of neurons with diversity(non-identical neurons), it is very important to investigate the consensus of collective behaviors thus synchronization is discussed on neuronal networks [32–34]. In fact, the collective behaviors and synchronization transition of network could be dependent on the local kinetics of the nodes, and it is important to find reliable neuron model to describe the local kinetics of the neuronal network. The author of this paper argued that the effect of electromagnetic induction should be considered during the fluctuation of inter-cellular and extra-cellular ion concentration because time-varying electromagnetic field can be triggered and set up. On the other hand, the exchange of ion channel current can also change the distribution of electromagnetic field in the media. Therefore, magnetic flux and memristor are used to develop a more reliable neuron model [35]. It is interesting to find electromagnetic radiation can induce multiple modes of electrical activities [36], and also phase synchronization and double coherence resonance [37] under phase noise. Indeed, due to diversity of realistic systems, phase synchronization [38] between coupled oscillators and/neurons could be more acceptable than complete synchronization. Within the synchronization problems, conditional Lyapunov exponents are calculated to find the threshold for coupling intensity. For two identical oscillators or neurons, appropriate coupling intensity can realize complete synchronization and phase synchronization. For non-identical oscillators, phase synchronization becomes available under coupling, and the Largest Lyapunov exponent and Lyapunov dimension given by Kaplan–Yorke formula with respect to finite time Lyapunov exponents are often calculated to detect the synchronization approaching. Researchers prefer to using the standard algorithm proposed by wolf [39], and some researchers [40,41] gave further feasible discussion about the calculating about Lyapunov exponents, which show some difference by setting different initials even the parameters are fixed.

Synapse connection is effective to propagate signals between neurons, the chemical or electrical synapse can make neuron become excitatory or inhibitory as well. In fact, the neuronal system consists of a large number of neurons and complete synchronization or being behavior consensus could be difficult due to the diversity of neuron, it is interesting to find another effective way for signal communication between neurons. Indeed, rhythm and phase synchronization can be effective for signal exchange and information encoding between neurons. Besides the synapse coupling, there could be other ways to realize signals between neurons. In this paper, we suggested that neurons can exchange signals by setting different electromagnetic field and magnetic flux coupling could be available in physical view.

2. Model description and scheme

Considered the effect of electromagnetic induction, magnetic flux is introduced into the previous Hindmarsh–Rose neuron model, and the electrical activities of isolate neuron will be described by the four-variable dynamical equations [35,36]. Two neurons are coupled by magnetic flux, and the dynamical equations for the drive and response systems can be described as follows

$$\begin{cases} \dot{x}_1 = y_1 - a{x_1}^3 + b{x_1}^2 - z_1 + I_{ext} - k\rho(\varphi)x_1 \\ \dot{y}_1 = c - d{x_1}^2 - y_1 \\ \dot{z}_1 = r[s(x_1 + 1.6) - z_1] \\ \dot{\varphi}_1 = k_1 x_1 - k_2\varphi_1 + D(\varphi_2 - \varphi_1) \end{cases} \tag{1}$$

$$\begin{cases} \dot{x}_2 = y_2 - a{x_2}^3 + b{x_2}^2 - z_2 + I_{ext} - k\rho(\varphi)x_2 \\ \dot{y}_2 = c - d{x_2}^2 - y_2 \\ \dot{z}_2 = r[s(x_2 + 1.6) - z_2] \\ \dot{\varphi}_2 = k_1 x_2 - k_2\varphi_2 + D(\varphi_1 - \varphi_2) \end{cases} \tag{2}$$

where x, y, z, φ describes the membrane potential, recovery variable for slow current and adaption current, respectively. I_{ext} is the external forcing current, and a,b,c,d,r,s are parameters, k,k_1,k_2 are feedback gains, φ is the magnetic flux across