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The 2-good-neighbor (2-extra) diagnosability of alternating group graph networks under the PMC model and MM* model☆

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ABSTRACT

Diagnosability of a multiprocessor system is one important study topic. In 2012, Peng et al. proposed a measure for fault tolerance of the system, which is called the g-good-neighbor diagnosability that restrains every fault-free node containing at least g fault-free neighbors. In 2016, Zhang et al. proposed a new measure for fault diagnosis of the system, namely, the g-extra diagnosability, which restrains that every fault-free component has at least $(g+1)$ fault-free nodes. As a favorable topology structure of interconnection networks, the n-dimensional alternating group graph network AN_n has many good properties. In this paper, we obtain that (a) the 2-good-neighbor diagnosability of AN_n is $3n-7$ for $n \geq 4$ under the PMC model and MM* model; (b) the 2-extra diagnosability of AN_n is $3n-7$ for $n \geq 4$ under the PMC model, and the 2-extra diagnosability of AN_n is $3n-7$ for $n \geq 5$ under the MM* model. These results are optimal with respect to 2-extra diagnosability of AN_n.

1. Introduction

Many multiprocessor systems have interconnection networks (networks for short) as underlying topologies. A network is usually represented by a graph where nodes represent processors and links represent communication links between processors. So we use graphs and networks interchangeably. For the system, study of the topological properties of its network is important. Furthermore, some processors may fail in the system, so processor fault identification plays an important role for reliable computing. The first step to deal with faults is to identify the faulty processors from the fault-free ones. The identification process is called the diagnosis of the system. A system is said to be t-diagnosable if all faulty processors can be identified without replacement, provided that the number of faults presented does not exceed t. The diagnosability $t(G)$ of a system G is the maximum value of t such that G is t-diagnosable [6,7,11]. For a t-diagnosable system, Dahbura and Masson [6] proposed an algorithm with time complex $O(n^{2.5})$, which can effectively identify the set of faulty processors.

Several diagnosis models were proposed to identify the faulty processors. One major approach is the Preparata, Metze, and Chien's (PMC) diagnosis model introduced by Preparata et al. [15]. The diagnosis of the system is achieved through two linked processors testing each other. Another major approach is the Maeng and Malek's (MM) diagnosis model, namely the comparison diagnosis model, proposed by Maeng and Malek [13]. In the MM model, to diagnose a system, a node sends the

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same task to two of its neighbors, and then compares their responses. Sengupta and Dahbura [17] proposed a special case of the MM model, called the MM* model, in which each node must test its any pair of adjacent nodes. The PMC model and MM* model have been widely used in the pioneering work. In 2005, Lai et al. [11] introduced a restricted diagnosability of a multiprocessor system called conditional diagnosability. They considered the situation that no fault set can contain all the neighbors of any vertex in the system. In 2012, Peng et al. [14] proposed a measure for fault diagnosis of systems, namely, the g-good-neighbor diagnosability (which is also called the g-good-neighbor conditional diagnosability), which requires that every fault-free node has at least g fault-free neighbors. In [14], they studied the g-good-neighbor diagnosability of the n-dimensional hypercube under the PMC model. In 2016, Wang and Han [18] studied the g-good-neighbor diagnosability of the n-dimensional hypercube under the MM* model. In 2016, Xu et al. [23] studied the g-good-neighbor diagnosability of complete cubic networks under the PMC model and MM* model. Yuan et al. [24,25] studied that the g-good-neighbor diagnosability of the k-ary n-cube ($k \geq 3$) under the PMC model and MM* model. As a favorable topology structure of interconnection networks, the Cayley graph $C\Gamma_n$ generated by the transposition tree Γ_n has many good properties. In [19,20], Wang et al. studied the g-good-neighbor diagnosability of $C\Gamma_n$ under the PMC model and MM* model for $g = 1, 2$. In 2016, Zhang and Yang [26] proposed a new measure for fault diagnosis of the system, namely, the g-extra diagnosability, which restrains that every fault-free component has at least $(g+1)$ fault-free nodes. In [26], they studied the g-extra diagnosability of the n-dimensional hypercube under the PMC model and MM* model. In 2016, Wang et al. [21] studied the 2-extra diagnosability of the bubble-sort star graph BS_n under the PMC model and MM* model. In 2017, Wang et al. [22] determined that the 2-good-neighbor connectivity and 2-good-neighbor diagnosability of BSn. As a favorable topology structure of interconnection networks, the n-dimensional alternating group graph network AN_n has many good properties. In this paper, it is proved that (a) the 2-extra connectivity of AN_n is $3n-9$ and this result is optimal; (b) the 2-good-neighbor diagnosability of AN_n is $3n-7$ for $n \geq 4$ under the PMC model and MM* model; (c) the 2-extra diagnosability of AN_n is $3n-7$ for $n \geq 4$ under the PMC model, and the 2-extra diagnosability of AN_n is $3n-7$ for $n \geq 5$ under the MM* model. These results are optimal with respect to 2-extra diagnosability of AN_n.

2. Preliminaries

In this section, some definitions and notations needed for our discussion, the alternating group graph network, the PMC model and the MM* model are introduced.

2.1. Notations

A multiprocessor system is modeled as an undirected simple graph $G = (V, E)$, whose vertices (nodes) represent processors and edges (links) represent communication links. Given a nonempty vertex subset V' of V, the *induced subgraph* by V' in G, denoted by $G[V']$, is a graph, whose vertex set is V' and the edge set is the set of all the edges of G with both endpoints in V'. The *degree* $d_G(v)$ of a vertex v is the number of edges incident with v. The *minimum degree* of a vertex in G is denoted by $\delta(G)$. For any vertex v, we define the *neighborhood* $N_G(v)$ of v in G to be the set of vertices adjacent to v. u is called a neighbor vertex or a neighbor of v for $u \in N_G(v)$. Let $S \subseteq V$. We use $N_G(S)$ to denote the set $\cup_{v\in S} N_G(v) \setminus S$. For neighborhoods and degrees, we will usually omit the subscript for the graph when no confusion arises. A graph G is said to be k-regular if for any vertex v, $d_G(v) = k$. The *connectivity* $\kappa(G)$ of a connected graph G is the minimum number of vertices whose removal results in a disconnected graph or only one vertex left when G is complete. A complete graph of order n is denoted by K_n. A graph is bipartite if its vertex set can be partitioned into two subsets X and Y so that every edge has one end in X and one end in Y; such a partition (X, Y) is called a bipartition of the graph, and X and Y its parts. We denote a bipartite graph G with bipartition (X, Y) by $G = (X, Y; E)$. If $G = (X, Y; E)$ is simple and every vertex in X is joined to every vertex in Y, then $G = (X, Y; E)$ is called a complete bipartite graph, denoted by $K_{n,m}$, where $|X| = n$ and $|Y| = m$. Let F_1 and F_2 be two distinct subsets of V, and let the *symmetric difference* $F_1 \triangle F_2 = (F_1 \setminus F_2) \cup (F_2 \setminus F_1)$. Let $B_1, \ldots, B_k$ ($k \geq 2$) be the components of $G - F_1$. If $|V(B_1)| \leq \cdots \leq |V(B_k)|$ ($k \geq 2$), then B_k is called the maximum component of $G - F_1$. For graph-theoretical terminology and notation not defined here we follow [2].

Let $G = (V, E)$ be a connected graph. A fault set $F \subseteq V$ is called a *g-good-neighbor faulty set* if $|N(v) \cap (V \setminus F)| \geq g$ for every vertex v in $V \setminus F$. A *g-good-neighbor cut* of a graph G is a g-good-neighbor faulty set F such that $G - F$ is disconnected. The minimum cardinality of g-good-neighbor cuts is said to be the g-good-neighbor connectivity of G, denoted by $\kappa^{(g)}(G)$. A fault set $F \subseteq V$ is called a *g-extra faulty set* if every component of $G - F$ has at least $(g+1)$ vertices. A *g-extra cut* of G is a g-extra faulty set F such that $G - F$ is disconnected. The minimum cardinality of g-extra cuts is said to be the g-extra connectivity of G, denoted by $\tilde{\kappa}^{(g)}(G)$.

Proposition 2.1 [16]**.** *Let G be a connected graph. Then* $\tilde{\kappa}^{(g)}(G) \leq \kappa^{(g)}(G)$. *In particular,* $\kappa^{(1)}(G) = \tilde{\kappa}^{(1)}(G)$.

2.2. The PMC model and the MM* model

Under the PMC model [15], to diagnose a system G, two adjacent nodes in G have the capability of performing tests on each other. For two adjacent nodes u and v in $V(G)$, the test performed by u on v is represented by the ordered pair (u, v). The outcome of a test (u, v) is 1 (resp. 0) if u evaluates v as faulty (resp. fault-free). In the PMC model, we usually assume that the testing result is reliable (resp. unreliable) if the node u is fault-free (resp. faulty). A test assignment T for a system