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A fast memory-saving method for the Morlet wavelet-based transform and its application to *in vivo* assessment of microcirculation dynamics

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ABSTRACT

We propose an approach to the fast numerical evaluation of the continuous wavelet transform with the Morlet wavelet based on the replacement of the Gaussian factor in the latter by a hierarchical sequence of B-splines and their application to the image formed by the analyzed signal multiplied by a set of periodic functions. We show that the suggested approach results in a significant reduction of the resulting data storage (almost linear dependence on the sample length vs. the quadratic one required by conventional methods). In order to illustrate the workability of the proposed technique we applied it to the analysis of the data on the blood flow velocity data obtained in vivo from chorioallantoic membrane of chicken embryo, which exhibit non-stationarity, bi- or multirhythmicity.

1. Introduction

Among various fields of application, a special permanent interest to the continuous wavelet transform (CWT) is fed by the need for processing of biophysical and biomedical data [1–4]. This type of data is characterized by a high variability and complexity, showing different coexisting oscillating non-linear modes, transient regimes, data noisiness, and by the requirement for their fast processing during in vivo experiments. In turn, all above provides a mathematical challenge for applied functional analysis and theory of numerical methods [5].

The measurement and quantification of blood flow in small vessels (microcirculation) is the target problem for many optical imaging methods [6,7]. Being different in specific techniques used, these methods typically provide the raw data in the form of recorded sequence of images, and the desired information might be obtained from the temporal and/or spatial variability, rather than from the value (brightness) of each pixel.

Often, not the whole image, but some selected region of interest (ROI), which represents the target object for measurement is the subject for further processing, especially if the image is of high-resolution and its post-processing is computationally expensive. Thus, different methods aimed to automatically select and highlight ROIs were developed, say, for separate visualization of arterial and venous vessels on the basis of mean flow rate [8].

The pulsative behavior of blood flow is one of the target objects of measurements [9], but potentially can also be used to distinguish between different areas and to suggest ROIs automatically. However, this approach implies “on-the-fly”

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detection and quantification of rhythms that appears to be a real challenge from the computational viewpoint. In our paper we propose the solution which is especially suitable for the fast search and assessment of rhythms using relatively short time series of 2^{10}–2^{11} points.

The core of proposed approach is the special method for a fast and memory-saving estimation of the continuous wavelet transform coefficients that makes it possible to apply it to the current batch of recorder frames.

The continuous wavelet transform is defined as the integral transform

$$w(a,b) = C(a)\int_{-\infty}^{+\infty} f(t)\psi^*\left(\frac{t-b}{a}\right)dt \tag{1}$$

with a self-similar sliding window ψ (the wavelet, the asterisk denotes complex conjugation), which produces two-dimensional functions, the arguments of which are the scale a (it describes the characteristic periods of the transformed function $f(t)$) and the shift b, which plays a role of the instant time. $C(a)$ defines a normalization.

From the point of view of period determination, of practical utility is the standard Morlet wavelet $\psi(\xi) = \exp(i\omega_0\xi - \xi^2/2)$ with the amplitude norm $C(a) = (2\pi a^2)^{1/2}$ [10] because of its special simplicity of expression that relates the scale of wavelet maxima, the central frequency ω_0, and the periods of oscillating components of the analyzed signal.

However, this choice (as well as others) results in computational complexity and high memory consumption in practical calculations: the kernel in Eq. (1) is an oscillating function, which needs to be coordinated with sampling of the transformed function $f(t_j)$. The numerical convolution for long samples and a large set of scales is slow and may require a plenty of storage in the case of its direct implementation [11,12].

For this reason, the most popular modern method for the CWT with the Morlet wavelet uses the Fast Fourier Transform (FFT) as an intermediate step, basing on the convolution theorem [13]. It is sufficiently fast and numerically stable but still requires a large memory storage for the results.

Whence, there exist approaches to overcome this difficulty. In principle, all of them attempt to reduce the continuous wavelet transform to the discrete wavelet transform (DWT), which can be evaluated via the application of a system of sparse filters. The pioneering works [14,15] simply apply the octave discretization of the shift and the scale as $a_j = 2^j$, $b = 2^j k$ (with respect to the unit step of sampling) and replace CWT by DWT with the filter resembling the original wavelet functions. But such a procedure results in gaps in output data, i.e. loses the principal advantage of continuity of the CWT. This difficulty was overcome by the fitting of both, the analyzed function and the Morlet wavelet, by B-splines and further processing by spline-based filters for continual distribution of scales [16] and by the introduction of a frequency dependent time shift [17].

On the other hand, these approaches still operate with various approximations of wavelets as oscillating functions, e.g. are influenced by the requirement of complex shape fitting, and provide an approximate covering the whole time interval by discrete functions (so called partition of unity problem). Thus, the first aim of the present work, is to resolve the difficulties mentioned above without a loss of time-saving and computational efficiency. The second one is an application of the algorithm under consideration to the real complex non-stationary data, which originate from the experiments of the study of microvascular dynamics in chicken embryo.

2. Method

2.1. CWT with the Morlet wavelet as an anisotropic diffusion smoothing

The continuous wavelet transform with the Morlet wavelet represented explicitly as

$$w(a,b) = \int_{-\infty}^{+\infty} f(t)e^{-i\omega_0\frac{(t-b)}{a}}e^{-\frac{(t-b)^2}{2a^2}}\frac{dt}{\sqrt{2\pi a^2}},$$

can be rewritten in the equivalent representation

$$w(a,b) = e^{i\frac{\omega_0}{a}b}\int_{-\infty}^{+\infty}\left[f(t)e^{-i\frac{\omega_0}{a}t}\right]e^{-\frac{(t-b)^2}{2a^2}}\frac{dt}{\sqrt{2\pi a^2}}. \tag{2}$$

Formally, the integral transform (2) has a form of the diffusion smoothing applied to the two-dimensional complex function $g(t,a) = f(t)\exp\left(-i\omega_0 a^{-1}t\right)$ evaluated along b-axis for a set of fixed scales a, i.e. anisotropically.

The demonstrative mechanism of this process, which leads to the extraction of local maxima, can be illustrated by the example of a function with a zero mean, locally expanded into the Fourier series

$$f(t) = \sum_n C_n e^{i\omega_n t}.$$

Then, the considered two dimensional function is

$$\tilde{f}(t,a) = C_{n'} + \sum_{n\neq n'} C_n e^{i\left(\omega_{n'}-\omega_0 a^{-1}\right)t}, \tag{3}$$

where n' is defined as a number of the Fourier coefficient, which fulfills the equality $\omega_{n'} - \omega_0 a^{-1} = 0$ for a fixed a.

As a result, the diffusion smoothing keeps the constant $C_{n'}$ unaffected and diminishes oscillating terms.