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Moment stability via resolvent operators of fractional stochastic differential inclusions driven by fractional Brownian motion

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ABSTRACT

In this manuscript, we consider a class of fractional stochastic differential inclusions driven by fractional Brownian motion in Hilbert space with Hurst parameter $\hat{H} \in (\frac{1}{2}, 1)$. Sufficient conditions for the existence and asymptotic stability of mild solutions are derived in mean square moment by employing fractional calculus, analytic resolvent operators and Bohnenblust–Karlin's fixed point theorem. The effectiveness of the obtained theoretical results is illustrated by an example.

1. Introduction

In recent years, fractional calculus has received much attention due to its excellence in describing real life phenomena in widespread science and engineering fields. Phenomena with memory and hereditary characteristics which arise in mechanics, electrical engineering, medicine, biology, and ecology, to name just a few, may be well modelled using fractional differential equations (FDEs) (see [19,31]).

The stability analysis of differential equations through its solutions is one of the core topic in mathematics. Lyapunov's direct method is an effective tool to analyze the stability of a wide class of ordinary differential equations (ODEs). However, there are large set of problems for which the Lyapunov direct method is ineffective. Moreover, Lyapunov's direct method is only a sufficient condition, which means if one cannot find a Lyapunov function candidate to conclude the system stability property, the system may still be stable and one cannot claim the system is not stable [23]. The stability analysis of this kind of challenging problem could be established by using fixed point theorem approach (see [2,33]).

The fractional derivatives are nonlocal and have weakly singular kernels, hence the stability analysis of FDEs is more complex than that of classical integer order differential equations. Different kind of approaches are introduced for the stability analysis of FDEs. Li et al. [24] proposed the definition of Mittag–Leffler stability and introduced the fractional Lyapunov direct method. Wang et al. [37] studied the stability analysis of FDEs by using comparison principle. The generalized Gronwall–Bellman inequality has been applied in [28] to study the stability of FDEs. The sufficient conditions for the exis-

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tence and asymptotic stability in pth moment of mild solutions were derived for fractional stochastic evolution equations with Poisson jumps by employing the Banach fixed point principle [39].

There is a wide range of interesting processes in robotics, control theory, economics, that can be described as a differential equations with non-deterministic dynamics. This is because in some situations modeling of natural phenomena by deterministic differential equations is not satisfactory, due to the existence of fluctuations in nature. Much information is available in the literature for the existence, controllability and stability analysis of stochastic differential equations (SDEs) (see [4,5,32]). However, the investigation on stability analysis of fractional stochastic differential equations/inclusion (FSDEs/FSDIs) remains in the initial stage with much work remaining to be done.

The noises arise in mathematical finance, communication networks, hydrology and medicine etc., are well modeled by fractional Brownian motion (fBm) (see [6,10,18,27]). Thus the SDEs with fBm have been considered greatly by research community in various aspects (see [9,15]). The FSDEs driven by fBm have been considered in the literature due to its salient features for theory and applications alike. Li [22] obtained the sufficient conditions for the existence and uniqueness of mild solutions for stochastic delay fractional evolution equations driven by fBm. Guendouzi and Idrissi [17] investigated the approximate controllability of fractional stochastic functional evolution equations driven by a fBm with Hurst parameter $H > \frac{1}{2}$.

The differential equation with multivalued right hand side is called differential inclusion. Differential inclusions possessing additional random terms are termed to be stochastic differential inclusions (SDIs). Likewise FSDI is generalization of FSDE through multivalued analysis. Owing to the real world applications of SDEs/SDIs driven by fBm namely in the fields financial markets and traffic networks etc. (see [21,25,29]), it is crucial to consider qualitative properties for FSDIs driven by fBm.

For a class of FDEs/FSDEs of order $1 < \alpha < 2$, dos Santos et al. [13] studied the existence of mild solutions for abstract fractional neutral integro-differential equations with state-dependent delay by using the Leray–Schauder alternative fixed point theorem. Ge and Kou [16] studied the stability analysis of nonlinear FDEs of order $1 < \alpha < 2$ by using Krasnoselskii's fixed point theorem. For some notable contributions in this direction, one can refer [1,14].

Further in the case of FDIs/FSDIs of order $1 < \alpha < 2$, the existence, controllability, stability analysis and other qualitative and quantitative properties have not received much attention from researchers (see [35,36,38]). In particular to the best of authors knowledge, stability analysis of FDIs of order $1 < \alpha < 2$ is as yet untreated. The main objective of this manuscript is to investigate the asymptotic stability by using a fixed point theorem for the FSDIs driven by fBm of the following form

$$ {}^cD_t^\alpha x(t) \in Ax(t) + \int_0^t B(t-s)x(s)ds + F(t,x(t)) + \sigma(t)\frac{dB^{\hat{H}}(t)}{dt}, \ \ t \in J = [0,b] \tag{1} $$

$$ x(0) = x_0, \ \ x'(0) = 0 \tag{2} $$

where $\alpha \in (1, 2)$, the closed linear operators A, $(B(t))_{t\,\geq\,0}$ are defined on a common domain, which is dense in a Hilbert space H. $x(\cdot)$ takes values in the separable Hilbert space H with inner product $\langle\cdot,\cdot\rangle_H$ and norm $\|\cdot\|_H$. Let K be another separable Hilbert space with inner product $\langle\cdot,\cdot\rangle_K$ and norm $\|\cdot\|_K$. $B^{\hat{H}}$ is a fBm with the Hurst parameter $\hat{H} \in (\frac{1}{2}, 1)$. The function F: $J \times H \to 2^H/\{\emptyset\}$ is a nonempty, bounded, closed and convex multivalued map and $\sigma : J \to L_2^0(K,H)$ is a deterministic function, where $L_2^0(K,H)$ denote the space of all Q-Hilbert–Schmidt operators from K to H. Let $\{B^{\hat{H}}(t)\}_{t\geq 0}$ be Q-fBm with Hurst index $\hat{H} \in (\frac{1}{2}, 1)$ defined in a complete probability space $(\Omega, \mathcal{F}, \{\mathcal{F}\}_{t\geq 0}; \mathbb{P})$ with values in a Hilbert space K.

This paper is organized as follows. In Section 2, we give some preliminaries, basic definitions and results, which will be used in the sequel. Section 3 is devoted to the proof of the existence and asymptotic stability of mild solutions for the considered system (1)–(2). In Section 4, we illustrate the derived theoretical results through an example. Conclusions are drawn and the direction of future work described in Section 5.

2. Preliminaries

In this section the essential basic preliminaries, definitions, notations and lemmas which are needed to establish the main results are presented. Let H, K represent real separable Hilbert spaces and $(\Omega, \mathcal{F}, \{\mathcal{F}\}_{t\geq 0}; \mathbb{P})$ is a complete probability space with natural filtration $\{\mathcal{F}_t\}_{t\geq 0}$, where $\mathcal{F}_t$ the σ-algebra is generated by random variable $\{B^{\hat{H}}(s), \ s \in [0,t]\}$ and $\mathbb{P}$-null sets. Let $L(K, H)$ denote the space of bounded linear operators from K to H. For convenience, the same notation $\|\cdot\|$ is used to denote the norms in H, K and $L(K, H)$ and the inner product of H, K is denoted by $\langle\cdot,\cdot\rangle$. Let $C = C(J,H)$ denote the family of continuous H-valued stochastic processes $\{\zeta(t), t \in J\}$ which are $\mathcal{F}_t$-measurable and $\|\zeta\| < \infty$, where

$$ \|\zeta\| = \|\zeta\|_C = \sup_{t\in J}\left(E\|\zeta\|^2\right)^{\frac{1}{2}}. $$

It is easy to verify that C, furnished with the norm topology as defined above, is a Banach space.

Definition 2.1. A one dimensional fBm $\{\beta^{\hat{H}}(t), \ t \geq 0\}$ with Hurst parameter $\hat{H} \in (0,1)$ is a zero mean Gaussian process with continuous sample paths such that

$$ R_{\hat{H}}(t,s) = E[\beta^{\hat{H}}(t)\beta^{\hat{H}}(s)] = \frac{1}{2}(t^{2\hat{H}} + s^{2\hat{H}} - |t-s|^{2\hat{H}}). $$