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Exponential synchronization of complex-valued complex networks with time-varying delays and stochastic perturbations via time-delayed impulsive control

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ABSTRACT

Considering the fact that time delays are unavoidable in the control of practical systems, this paper considers globally exponential synchronization of complex-valued complex dynamical networks with multiple time-varying delays and stochastic perturbations by designing a time-delayed impulsive control scheme. By taking the advantage of Lyapunov method in complex field and utilizing an impulsive inequality with delays, several synchronization criteria are obtained through strict mathematical proofs. Our results are general which extend some existing ones concerning impulsive synchronization. A numerical example is given to illustrate the effectiveness of theoretical results.

1. Introduction

With the development of science and technology, complex dynamical networks (CDNs) are becoming increasingly important in modern society. Up to date, a lot of researchers from different fields are interested in studying dynamics of CDNs such as physics, mathematics, secure communication, engineering, automatic control, biology and sociology [1–6]. Synchronization is one of the most important phenomenon for CDNs [7–19]. In [7], authors considered H_∞ synchronization performance of a nonlinear coupled network. Synchronization for a class of CDNs with disturbances was studied in [9] by designing distributed output-feedback protocol. Authors in [17] investigated nonfragile exponential synchronization of CDNs by designing memory sampled-data control. Authors in [18] studied finite-time stochastic synchronization problem for CDNs with stochastic noise perturbations. Note that the systems in the above mentioned references are all real-valued.

Recently, many chaotic systems with complex variables have been proposed, for instance, complex-valued Lorenz system, complex-valued neural networks [20–25]. Neural networks have been extensively investigated since their wide applications in different areas such as signal processing, moving image processing, optimization [26]. In [20], the stability problem for delayed complex-valued recurrent neural networks was considered. By separating complex-valued neural networks into real and imaginary parts and forming an equivalent real-valued system, Wu et al. [22] investigated complex projective synchronization in coupled dynamical systems with complex variables via feedback controller. Liu et al. [23] investigated robust adaptive full-state hybrid synchronization of chaotic complex-valued systems with unknown parameters and nonidentical external disturbances. Combination synchronization of three chaotic systems with complex variables was investigated in

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[24]. Drive-response synchronization for a class of complex-variable chaotic systems with uncertain parameters was studied in [25] via adaptive and impulsive controls. At the same time, in real world, signals transmitted between subsystems of CDNs are unavoidably subject to stochastic perturbations from environment uncertainties, which may cause losses of useful information. Therefore, it is very necessary to study the complex-valued system with stochastic perturbations. However, synchronization of complex-valued CDNs with stochastic perturbations was not considered in [20–25]. Moreover, To the best of our knowledge, few published papers consider synchronization of complex-valued CDNs with stochastic perturbations via impulsive control.

Impulsive effects are characterized by abrupt changes in the state of systems at certain instants which may be caused by switching phenomenon, frequency changes or other sudden noise [19,27–30]. On the other hand, impulsive effects have dual nature, *i.e.*, desynchronizing impulses [19,29,30] and synchronizing impulses [29,32]. Synchronizing impulses are often utilized to design impulsive control because they have some advantages that continuous controllers do not have. For instance, they need small impulsive gains and act only at discrete time instants. Therefore, impulsive control can reduce control cost and the amount of transmitted information drastically. In recent years, there are many published papers concerning synchronization and stability of CDNs via impulsive control scheme [19,27,31–33]. However, as is well known, due to limited transmission speed and bandwidth of channel, time delays in practical systems are unavoidable and cannot be ignored. Discarding time-delay in studying dynamical behaviors of practical systems may lead to wrong conclusion. In the literature, time delays in couplings and dynamical nodes have received considerable attention [19,22]. Actually, delays in impulses are also exist due to the finite speed of signal transmission [35–37]. Moreover, receiving signal from the transmitter and transmitting it from the controller to controlled system also need some time. Hence, it is necessary to consider time delay when impulsive control scheme is utilized to synchronize chaotic systems [33–35]. In the literature, stability and synchronization via delayed impulsive control have been investigated for real-valued systems [33–35]. However, to the best of our knowledge, no published paper consider synchronization or stability of complex-valued systems by using delayed impulsive control.

Motivated by the above discussions, this paper aims to investigate globally exponential synchronization of complex-valued CDNs with multiple time-varying delays and stochastic perturbations via time-delayed impulsive control. A set of time-delayed impulsive controllers are designed such that every node in it can be synchronized onto an isolated trajectory. Based on a special impulsive inequality in [35], sufficient conditions guaranteeing the exponential synchronization are derived by using Lyapunov method. Results of this paper are general and extend existing ones to complex field. Numerical simulations are provided to show the effectiveness of our results.

The rest of this paper is organized as follows. In Section 2, the considered model of complex-valued CDNs with stochastic perturbations and time-delayed impulsive control is proposed. Some necessary assumptions and lemmas are also given in this section. In Section 3, synchronization of the delayed CDNs is studied. Then, in Section 4, numerical simulations are given to demonstrate the effectiveness of our results. Finally, in Section 5, conclusions and future works are given.

Notations : the notations are quite standard. Throughout this paper, $\mathbb{R}$ and $\mathbb{C}$ denote the set of real and complex numbers, respectively. $\mathbb{R}^n$and $\mathbb{C}^n$ denote the set of the n-dimensional real-vector space and the n-dimensional complex-vector space, respectively. $\mathbb{R}^+$ and $\mathbb{R}^{n\times n}$ denote the set of nonnegative real numbers and $n\times n$ real matrix. The superscript T denotes transposition of matrices or vectors. The superscript $(\cdot)^*$ denotes conjugate transpose. I_n is the $n\times n$ identity matrix. For $z=(z_1,z_2,\ldots,z_n)^T\in\mathbb{C}^n$, $\|z\|=\sqrt{z^*z}$. $\lambda_{\max}(A)$ means the largest eigenvalue of a symmetric matrix A. $\mathbb{N}_+$ stands for the set of positive integers. Let $(\Omega,\mathcal{F},\{\mathcal{F}_t\}_{t\geq 0},P)$ be a complete probability space with filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions (*i.e.*, the filtration contains all P-null sets and is right continuous). Denote by $L^p_{\mathcal{F}_0}([-\tau,0];\mathbb{R}^n)$ the family of all $\mathcal{F}_0$-measurable $C([-\tau,0];\mathbb{R}^n)$-valued random variables $\xi=\{\xi(s):-\tau\leq s\leq 0\}$ such that $\sup_{-\tau\leq s\leq 0}\mathbf{E}\{\|\xi(s)\|^p\}<\infty$, where $\mathbf{E}\{\cdot\}$ stands for mathematical expectation operator with respect to the given probability measure P. Sometimes, the arguments of a function or a matrix will be omitted in the analysis when no confusion arise.

2. Model description and preliminaries

Consider a CDN consisting of N identical nodes with delays and stochastic perturbations as follows:

$$\begin{aligned}\mathrm{d}x_i(t) = &\Big\{f(t,x_i(t),x_i(t-\tau_1(t))) + \sum_{j=1}^{N} a_{ij}\Gamma x_j(t) + \sum_{j=1}^{N} b_{ij}\Gamma x_j(t-\tau_2(t))\Big\}\mathrm{d}t\\ &+\sigma_i(t,x(t),x(t-\tau_1(t)),x(t-\tau_2(t)))\mathrm{d}\omega_i(t),\ \ i=1,2,\ldots,N,\end{aligned}\tag{1}$$

where $x_i(t)=(x_{i1}(t),x_{i2}(t),\ldots,x_{in}(t))^T\in\mathbb{C}^n$ represents the state vector at time t; $\tau_l(t), l=1,2$ are time-varying delays satisfying $0<\tau_l(t)<\tau$, where τ is a positive constant; $f(t,x_i(t),x_i(t-\tau_1(t)))=(f_1(t,x_i(t),x_i(t-\tau_1(t))),\ldots,f_n(t,x_i(t),x_i(t-\tau_1(t))))^T\in\mathbb{C}^n$ is a continuous function; $\Gamma=\mathrm{diag}(\gamma_1,\gamma_2,\ldots,\gamma_n)$ with $\gamma_i>0$ $(i=1,2,\ldots,n)$ is the inner coupling matrix; $a_{ij}\in\mathbb{R}$ is defined as follows: if there is a connection from node j to node i $(i\neq j)$, then $a_{ij}>0$; otherwise $a_{ij}=0$, $a_{ii}=-\sum_{j=1,j\neq i}^{N}a_{ij}$. $b_{ij}\in\mathbb{R}$ is delayed outer couplings of the whole network, which satisfies the following conditions: $b_{ij}\neq 0$ and $b_{ii}=-\sum_{j=1,j\neq i}^{N}b_{ij}$. $\sigma_i(t,x(t),x(t-\tau_1(t)),x(t-\tau_2(t)))=\sigma_i(t,x_1(t),\ldots,x_n(t),x_1(t-\tau_1(t)),\ldots,x_n(t-\tau_1(t)),x_1(t-\tau_2(t)),\ldots,x_n(t-\tau_2(t)))\in\mathbb{R}^{n\times n}$ represents the perturbation strength, and $\omega_i(t)=(\omega_{i1}(t),\omega_{i2}(t),\ldots,\omega_{in}(t))^T\in\mathbb{R}^n$ is a bounded vector-form Weiner process.