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Random attractors for the coupled suspension bridge equations with white noises

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ABSTRACT

The paper is devoted to the investigation of the existence of a compact random attractor for the random dynamical system generated by the coupled suspension bridge equations with white noises.

1. Introduction

We consider the long-time behavior of the nonlinear dynamical system which describes the vibrating beam equation coupled with a vibrating string equation

$$\begin{cases} u_{1tt} + \alpha\Delta^2 u_1 + \delta_1 u_{1t} + k(u_1 - u_2)^+ + f_1(u_1) = q_1(x)\dot{W}_1, & \text{in } \Omega \times [\tau, +\infty), \ \tau \in \mathbb{R}, \\ u_{2tt} - \beta\Delta u_2 + \delta_2 u_{2t} - k(u_1 - u_2)^+ + f_2(u_2) = q_2(x)\dot{W}_2, & \text{in } \Omega \times [\tau, +\infty), \ \tau \in \mathbb{R} \end{cases} \tag{1.1}$$

with the simply supported boundary conditions at both ends

$$\begin{aligned} &u_1(x,t) = \Delta u_1(x,t) = 0, \quad x \in \partial\Omega, \ t \geq \tau, \\ &u_2(x,t) = 0, \quad x \in \partial\Omega, \quad t \geq \tau, \end{aligned} \tag{1.2}$$

and the initial-value conditions

$$\begin{aligned} &u_1(x,\tau) = u_{10}(x), \quad u_{1t}(x,\tau) = u_{11}(x), \ x \in \Omega, \\ &u_2(x,\tau) = u_{20}(x), \quad u_{2t}(x,\tau) = u_{21}(x), \ x \in \Omega. \end{aligned} \tag{1.3}$$

The first equation of (1.1) represents the vibration of the road bed in the vertical direction and the second equation describes that of the main cable from which the road bed is suspended by the tie cables (see [1,2]). Where $\Omega = [0, \ L]$, $k > 0$ denotes the spring constant of the ties, $\alpha > 0$ and $\beta > 0$ are the flexural rigidity of the structure and coefficient of tensible strength of the cable, respectively. $\delta_1, \delta_2 > 0$ are the damping coefficients, $u_i = u_i(x,t)$ $(i = 1, 2)$ are the real-valued functions on $\Omega \times [\tau, +\infty)$, and $\dot{W}_i(t)$ $(i = 1, 2)$ are the scalar Gaussian white noises, i.e., formally the derivative of the two-sided real-value scalar Wiener processes $\{W_i(t)\}_{t\in\mathbb{R}}$ $(i = 1, 2)$. Besides, for every $u \in \mathbb{R}$, $u^+ = \max\{u, 0\}$.

We assume that the functions $f_i, \ q_i$ $(i = 1, 2)$ always satisfy the following assumptions.

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(i) The nonlinear terms $f_1 \in C^3(\mathbb{R}, \mathbb{R})$ and $f_2 \in C^2(\mathbb{R}, \mathbb{R})$ with $f_1(0) = f_2(0) = 0$, which satisfy the following conditions.

$$|f_i(s)| \le C_0(1 + |s|^p), \ \ p \ge 1, \ \ \forall s \in \mathbb{R}, \ \ i = 1, 2, \tag{1.4}$$

where C_0 is a positive constant.

$$F_i(s) := \int_0^s f_i(r)dr \ge C_1(|s|^{p+1} - 1), \ \ p \ge 1, \ \ \forall s \in \mathbb{R}, \ \ i = 1, 2 \tag{1.5}$$

$$sf_i(s) \ge C_2(F_i(s) - 1), \ \ \forall s \in \mathbb{R}, \ \ i = 1, 2, \tag{1.6}$$

where C_1 and C_2 are positive constants.

(ii) $q_1(x) \in H^3(\Omega) \cap H_0^1(\Omega)$ and $q_2(x) \in H^2(\Omega) \cap H_0^1(\Omega)$ are not identically equal to zero.

The linear model (1.1) was originally introduced in [1] by Lazer and McKenna as a new problem in fields of the nonlinear analysis when the effect of white noises was neglected. In [2], Ahmed and Harbi proved the existence and uniqueness of the weak solution for the Cauchy problem of the deterministic coupled suspension bridge equations. Similar models have also been presented and considered by other some authors, but most of them were concentrated on either existence and decay estimates of solutions, or the approximations and numerical simulations, see [3,4,9–11] and references therein. We first investigated the existence of global attractors for both the single and the coupled deterministic suspension bridge equations using the methods of the energy estimates from the infinite dimensional dynamical system view, please refer the reader to [5,12,13,15,16]. Moreover, in the past decades, the long-time behavior of solutions for the deterministic suspension bridge equations have been extensively studied by several authors, for example, see also [6–8,14,17,28,30,31].

From the above presentation we can see that the universal attractors for the deterministic suspension bridge equations are better well investigated. To the best of our knowledge, the existence of the random attractors for the stochastic coupled suspension bridge equations are still not considered, while it is just our concerned.

Crauel and Flandoi et al. [18,19] originally introduced the random attractors for the infinite-dimensional random dynamical system(RDS). A random attractor of RDS is a measurable and compact invariant random set attracting all orbits. It is the appropriate generalization of the now classical attractor from the deterministic dynamical systems to the RDS. The reason is that if such a random attractor exists, it is the smallest attracting compact set and the largest invariant set [20]. These abstract results have been successfully applied into many stochastic dissipative partial differential equations such as reaction diffusion equations, Navier–Stokes equations and nonlinear wave equations etc., see [21–25] and references therein. For instance, Fan [21] proved the existence of random attractor for a damped sine-Gordon equation, Yang et al. [23,24] studied random attractors for the stochastic semi-linear degenerate parabolic equation and the random wave equation with nonlinear damping and white noise, respectively. Recently, Ma and Xu obtained the existence of the random attractor for the single stochastic suspension bridge equation with white noise in [27,29].

Similar to the deterministic systems, the key step in proving the existence of attractors for the random dynamical systems is to verify the compactness of the system in some sense. In fact, for the stochastic case, there does also exist compact invariant sets, however, they are not fixed, but depend on chance, and move with time. Therefore, establishing a compact random invariant set is our main task in this paper. With the help of the methods and techniques used in [18,23], we prove the existence of random attractors for the problem (1.1)–(1.3).

The outline of this paper is as follows: Background materials on RDS and random attractors are iterated in Section 2. In Section 3, we present the existence and uniqueness of the solution corresponding to the system (1.1)-(1.3) which determines a RDS. Finally, the existence of random attractors is shown in Section 4.

2. Random dynamical system

In this section, we recall some basic concepts related to the RDS and random attractor for a RDS in [18–20], which are important for getting our main results.

Let $(X, \|\cdot\|_X)$ be a separable Hilbert space with Borel σ-algebra $\mathcal{B}(X)$, and $(\Omega, \mathcal{F}, P)$ be a probability space. $\theta_t : \Omega \to \Omega, t \in \mathbb{R}$ is a family of measure preserving transformations such that $(t, \omega) \mapsto \theta_t\omega$ is measurable, θ_0=id and $\theta_{t+s} = \theta_t\theta_s$ for all $t, s \in \mathbb{R}$. The flow θ_t together with the probability space $(\Omega, \mathcal{F}, P, (\theta_t)_{t\in\mathbb{R}})$ is called as a metric dynamical system.

Definition 2.1. Let $(\Omega, \mathcal{F}, P, (\theta_t)_{t\in\mathbb{R}})$ be a metric dynamical system. Suppose that the mapping $\phi : \mathbb{R}^+ \times \Omega \times X \to X$ is $(\mathcal{B}(\mathbb{R}^+) \times \mathcal{F} \times \mathcal{B}(X), \mathcal{B}(X))$-measurable and satisfies the following properties:

(i) $\phi(0, \omega)x = x$, $x \in X$ and $\omega \in \Omega$;
(ii) $\phi(t + s, \omega) = \phi(t, \theta_s\omega) \circ \phi(s, \omega)$ for all $t, s \in \mathbb{R}^+$, $x \in X$ and $\omega \in \Omega$.

Then ϕ is called a random dynamical system (RDS). Furthermore, ϕ is called a continuous RDS if ϕ is continuous with respect to x for $t \ge 0$ and $\omega \in \Omega$.

Definition 2.2. A set-valued map D: $\Omega \to 2^X$ is said to be a closed (compact) random set if $D(\omega)$ is closed (compact) for P-a.s. $\omega \in \Omega$, and $\omega \mapsto d(x, D(\omega))$ is P-a.s. measurable for all $x \in X$.