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Steady heat conduction analyses using an interpolating element-free Galerkin scaled boundary method

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ABSTRACT

Through use of the improved interpolating moving least-squares (IIMLS) shape functions in the circumferential direction of the scaled boundary method based on the Galerkin approach, an interpolating element-free Galerkin scaled boundary method (IEFG–SBM) is developed in this paper for analyzing steady heat conduction problems, which weakens the governing differential equations in the circumferential direction and seeks analytical solutions in the radial direction. The IIMLS method exhibits some advantages over the moving least-squares approximation and the interpolating moving least-squares method because its shape functions possess the delta function property and the involved weight function is nonsingular. In the IEFG–SBM, only a nodal data structure on the boundary is required and the primary unknown quantities are real solutions of nodal variables. Higher accuracy and faster convergence are obtained due to the increased smoothness and continuity of shape functions. Based on the IEFG–SBM, the steady heat conduction problems with thermal singularities and unbounded domains can be ideally modeled. Some numerical examples are presented to validate the availability and accuracy of the present method for steady heat conduction analysis.

1. Introduction

The analysis of heat conduction problems is very important to engineering and science. However, due to the intrinsic complexity of the corresponding governing equation, analytical solutions for such problems are restricted to simple geometries and boundary conditions. Therefore, more and more efforts have been devoted to analyzing heat conduction problems by utilizing finite element methods (FEM) [1] and boundary element methods (BEM) [2]. Nevertheless, in heat conduction problems with thermal singularity, reasonable accuracy is often impossible or, at least, very costly to obtain by standard numerical methods [3–5]. In order to obtain satisfactory accuracy for heat conduction problems with thermal singularities, special treatments have to be introduced, which may complicate the numerical implementation. Moreover, a difficulty in the application of the FEM for heat conduction problems involving unbounded domain [6] lies in the effective treatment of unbounded domain, because these cannot be subdivided into a finite number of elements. For this reason, it is still necessary and urgent to develop novel numerical methods for solving heat conduction problems with thermal singularities and unbounded domains.

The scaled boundary method, developed originally by Wolf and Song [7] in 1996, is an important and attractive computational method as it can represent singularities and unbounded domains accurately and efficiently. It is semi-analytical in the

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sense of satisfying the equilibrium requirement in the strong form in the radial direction [8]. In this method, a scaled boundary coordinate system is introduced, which scales the boundary to a single point called the scaling center. The governing partial differential equations of various linear problems are weakened in the circumferential direction and then analytical solutions are sought in the radial direction [8]. Thus, only the boundary of the domain is discretized, but no fundamental solution is required. Obviously, the computational accuracy of the scaled boundary method may be affected greatly by the discretization approach employed in the circumferential direction [9]. The FEM has been widely used for performing this circumferential discretization and the corresponding method is called the scaled boundary finite element method [10–14]. However, in the scaled boundary finite element method, meshing is still arduous, time consuming and fraught with pitfalls for some problems, especially three-dimensional complicated engineering problems.

As alternative numerical approaches to eliminate the well-known drawbacks in the mesh-based methods, meshless methods have received much attention and gained great success in the field of computational science and engineering. Based on different approximation functions, many kinds of meshless methods have been proposed. Some representative examples are the element-free Galerkin (EFG) method [15,16], the meshless local Petrov–Galerkin method (MLPG) [17,18], the reproducing kernel particle method (RKPM) [19] and the point interpolation method (PIM) [20]. Of these meshless methods, the EFG method is a Galerkin discretization technique with the help of shape functions constructed using the moving least-squares (MLS) approximation. To use the MLS approximation, only an array of nodes is required in the domain under consideration. The EFG method exhibits some advantages such as increasing accuracy and rate of convergence, requiring no post-processing to obtain a smooth gradient field [15,16]. In order to unite advantages of the scaled boundary method and the EFG method, a novel element-free Galerkin scaled boundary method (EFG–SBM) [21–23] is proposed through the incorporation of the EFG approach into the scaled boundary method. The EFG–SBM is a boundary-type meshless method because no mesh generation is necessary and only a nodal data structure on the boundary is required. It has been demonstrated [21–23] that the EFG–SBM performs better than the conventional scaled boundary finite element method. Compared with the MLPG scaled boundary method [24], the EFG–SBM can yield a symmetric stiffness matrix and a more robust solution by retaining the Galerkin approach. However, essential boundary conditions in the EFG–SBM cannot be imposed directly because the obtained shape functions from the MLS approximation lack, in general, the delta function property.

In order to restore the delta function property of the MLS approximation, an interpolating moving least-squares (IMLS) method was developed by Lancaster and Salkauskas [25], which employs specific singular functions as weight functions. In order to obtain higher computational efficiency, a simpler formula of the shape function in the IMLS method was further presented by Ren and Cheng [26–28]. However, the use of IMLS method still has great difficulty in numerical computation because the singularity involved in the weight function complicates the computation of the inverse of the singular matrix and thus increases the computational cost. To overcome this shortcoming, a perturbation technique was introduced by Netuzhylov [29] into the IMLS method. However, a new positive parameter required in this technique always affects the computational accuracy. This led Wang et al. [30,31] to develop an improved interpolating moving least-squares (IIMLS) method with a nonsingular weight function, which can avoid problems in both the MLS and IMLS schemes. Later, some studies have been conducted to explore the possible application in meshless method [32,33]. The IIMLS shape functions possess the delta function property and therefore essential boundary conditions can be directly imposed in the IIMLS-based meshless method. In comparison with the IMLS method presented by Lancaster and Salkauskas, the key advantage of the IIMLS method is that it does not require singular weight function and thus any weight function used in the MLS approximation can also be applied in the IIMLS method. In addition, there are less unknown coefficients in the IIMLS method than in the MLS approximation. Thus fewer nodes are required in the local influence domain and higher computational accuracy can be reached in the IIMLS-based meshless method.

In this paper, an interpolating element-free Galerkin scaled boundary method (IEFG–SBM) is developed to solve steady heat conduction problems. The shape and test functions in the circumferential direction are constructed by the IIMLS method, which requires only a set of scattered nodes on the boundary. In order not to increase the smoothness of trial functions at corners, the IIMLS interpolation is independently implemented along each edge rather than the entire boundary. The IEFG–SBM is a direct boundary-type meshless method because the basic unknown quantities are the real solutions to the nodal variables and thus essential boundary conditions can be imposed directly. Besides, the scaled boundary equation in temperature is solved analytically using a matrix function solution technique [10]. Numerical examples presented in the end demonstrate that the proposed scheme for solving steady heat conduction problems behaves extremely well, even when the problem involves thermal singularities and unbounded domains.

2. Governing equations and boundary conditions

Consider a two-dimensional steady heat conduction problems for a stationary medium on a bounded domain Ω bounded by Γ. In the absence of heat generation, the governing differential equation can be written as follows

$$\nabla^2 T = 0 \text{ in } \Omega \tag{1}$$

with boundary conditions

$$T = \bar{T} \text{ on } S_1 \tag{2}$$

$$kT_{,i}n_i = \bar{q} \text{ on } S_2 \tag{3}$$