Applied Mathematics and Computation 301 (2017) 36–42

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

How does resolution of strategy affect network reciprocity in spatial prisoner's dilemma games?

Jun Tanimoto

Interdisciplinary Graduate School of Engineering Sciences, Kyushu University, Kasuga-koen, Kasuga-shi, Fukuoka 816-8580, Japan

ARTICLE INFO

Keywords:
Network reciprocity
Prisoner's dilemma
Evolutionary game

ABSTRACT

In the canonical framework of evolutionary 2×2 games, a binary strategy set comprising cooperation (C) and defection (D) has usually been presumed. Inspired by commonly observed real-world facts, we explore what happens if the resolution of strategy increases. As an extreme limit, the infinite resolution case is both a continuous and a mixed strategy defined by a real number in the range of [0,1]. We find that increasing resolution amplifies cooperation in spatial prisoner's dilemma games as compared with the binary strategy definition; however, this enhancement tendency with increasing resolution is not monotonic in the case of a mixed-strategy setting.

1. Introduction

For the past couple of decades, evolutionary game theory (EGT) has propelled our understanding of why and how human beings and other animal species evolved altruistic cooperation rather than selfish defection [1–3]. The prisoner's dilemma (PD) game, one of the four classes within 2×2 (two-player and two-strategy) games, wherein two agents independently decide between either cooperation (C) or defection (D), has provided a solid ground for this discussion. Meanwhile, network reciprocity, one of the five fundamental mechanisms for solving social dilemmas by adding "social viscosity" [4], has received intense attention [5–7] because although the central assumption of the model (namely, "playing with neighbors on an underlying network and copying a strategy from them") is simple, it still seems to be a plausible explanation for how cooperation enables survival in any real context (consult with text books [1–3] to understand what EGT is and how PD game meaningfully works to give a good metaphor for social dilemma in human society).

For the last decade, most works have concerned the question of what additional mechanism can bolster network reciprocity beyond that realized by the original model, i.e., the spatial prisoner's dilemma (SPD) game. Helped by the large stock of these previous studies and inspired by the fact that a clear view on the substantial discussion of SPD games is still lacking, we have posed the concepts termed END and EXP, which shed light on how network reciprocity is brought into evolutionary dynamics [8–13].

Incidentally, let us pose a simple question as to whether it is appropriate to assume the binary strategy set of C and D (hereafter called discrete strategy) in a real context. A human decision, or even a choice made by another animal species, seems more complex. The point of this argument is twofold; one is that there are more alternative choices than just two–C and D. The binary assumption, in a sense, limits the degree-of-freedom of the strategy space of [0,1] that originally contained any midway values ranging from complete defection ($D=0$) up to complete cooperation ($C=1$). Another point is that a choice is sometimes stochastic, rather than deterministic [14]. The discrete strategy system comprising C or D mentioned in

E-mail address: juntanimoto@gmail.com

http://dx.doi.org/10.1016/j.amc.2016.11.036

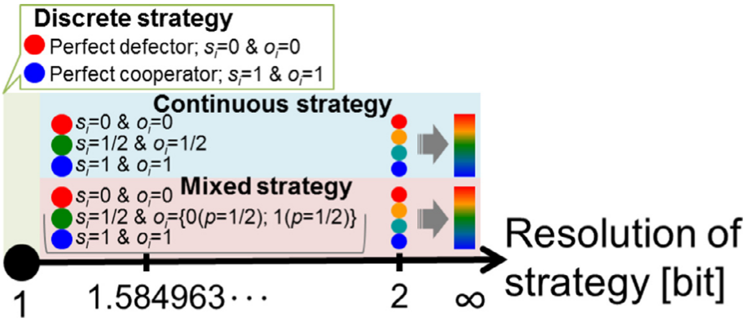

Fig. 1. Schematic expiation on how discrete, continuous. and mixed strategy systems differ from each other in relation to the resolution of strategy, where s_i and o_i respectively mean the strategy value and actual offer in a game of agent i, and p indicates the offering probability.

the above text is deterministic, where a game player i offers C ($o_i = 1$) when their strategy (s_i) is 1 and D ($o_i = 0$) when $s_i = 0$. The first point of the two comes from the fact that a real choice is often not all-or-nothing but halfway, allowing players to be "slightly" cooperative, "reasonably" defective, or neutral, for instance. The second point comes from the fact that a real choice (offer) based on a strategy sometimes deviates from the strategy because of stochastic perturbation. This can be reproduced by a so-called mixed strategy, where a player stochastically offers $o_i = \{D, C\}$ according to their strategy, s_i. There is another important strategy system that is the so-called continuous strategy, whereby a player offers the o_i that is completely consistent with their strategy s_i. A continuous strategy allows a player to make a halfway offer in [0,1]. Thus, the continuous strategy system is not stochastic but deterministic, despite the fact that its strategy value is defined by a real value like the mixed strategy system. Zhong et al. [15] and Kokubo et al. [16] have explored how the equilibria of continuous, mixed, and discrete strategy systems differ from each other in SPD games. Their works, however, were premised on the idea that the resolutions of both continuous and mixed strategy systems are infinite, allowing any halfway values in [0,1]. A natural question occurring to us is what happens if a finite resolution is presumed in the case of either a continuous or mixed strategy. For example, when three alternative choices are allowed instead of binary, the continuous strategy system enables $o_i = s_i = \{0, 1/2, 1\}$, while the mixed strategy system lets a player offer $o_i = \{D, C\}$ according to their strategy, $s_i = \{0, 1/2, 1\}$, meaning their C-offering probability (see Fig. 1). One plausible interest is whether network reciprocity can be fostered (or devastated) when the strategy resolution increases. The present paper makes a reply to this. Needless to say, a fine resolution brings a large information rate that can be quantified by a bit-value. The discrete strategy system, for example, contains only 1 bit where only either C or D can be offered. In the case of the three-alternative system mentioned above, the resolution comprises $\log_2 3 = 1.584963$ bits.

2. Model setup

As a matter of practice, it is difficult that any analytic approach gives a clear answer for the current question established in above section. Thereby, we take a simulation approach to obtain a practical answer.

At every time-step in the present model, each agent in the network (agent i) plays PD games with their immediate neighbors and obtains payoffs from all of them, which implies that an accumulated payoff is presumed. The underlying topology is a two-dimensional lattice graph (hereafter a lattice) with degree $k = 8$. The total number of agents is $N = 10^4$. After each time-step, each agent synchronously updates their strategy. In a PD game with discrete strategy, a player receives a reward (R) for each instance of mutual cooperation (C) in which they partake and a punishment (P) for each mutual defection (D). If one player chooses C and the other chooses D, the latter obtains a temptation payoff (T), and the former is labeled a sucker (S). Without loss of mathematical generality, a PD game space can be defined by presuming $R = 1$, $P = 0$, $S = -D_r$, and $T = 1 + D_g$, where D_g and D_r imply a chicken-type dilemma and a stag hunt-type (SH-type) dilemma, respectively [17,18]. Thus, the payoff matrix can be denoted by $\mathbf{M} = \begin{pmatrix} R & S \\ T & P \end{pmatrix} = \begin{pmatrix} 1 & -D_r \\ 1+D_g & 0 \end{pmatrix}$. As previously mentioned, PD is one of the four classes of 2×2 games and simultaneously contains a chicken-type dilemma and a SH-type dilemma, which are formulated by $0 < D_g$ and $0 < D_r$. In the following discussion, the PD game class is limited by assuming that $0 \le D_g \le 1$ and $0 \le D_r \le 1$.

Presuming the discrete strategy system, both an agent's strategy, s_i and their offer, o_i are consistent as either D (=0) or C (=1). When either a mixed or continuous strategy system is presumed, agent i has strategy s_i, denoted as a real number. Let us presume the partition number of the strategy space [0,1], n. The strategy resolution is defined by $\log_2 n$ [bit]. In the case of discrete strategy, it is presumed that $n = 2$, whereas both mixed and continuous strategies in the canonical meaning presume infinite n. In the present study, both mixed and continuous strategies presume $s_i = (m-1)/(n-1)$, where $m = 1, 2, \ldots, n$. In the case of mixed strategy, the offer o_i is stochastically decided as either D (=0) or C (=1) by the probability s_i. Meanwhile, in the case of a continuous strategy, an agent is allowed to offer s_i as their offer o_i. Thus, when agent i offering