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# $H_\infty$ control of Markov jump systems with time-varying delay and incomplete transition probabilities

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ABSTRACT

This paper addresses the $H_\infty$ control of continuous Markov jump systems with interval time-varying delay and incomplete transition probabilities. A linearization method is used to handle unknown transition probabilities. Meanwhile, the Wirtinger-based integral inequality and the reciprocally convex technique are adopted to deal with the time-varying delay. Additionally, a separating technique is employed to tackle the coupling among Lyapunov variable, system matrix and controller parameter. Based on these strategies, new sufficient conditions for the closed-loop system to be stochastically stable are formulated in the framework of linear matrix inequalities. Finally, numerical examples are provided to demonstrate the effectiveness of the proposed method.



## 1. Introduction

Markov jump systems (MJSs) belong to the category of stochastic hybrid systems with state and jump mode modeled by differential equations. Applications may be found in many processes, such as fault-tolerant systems, biology systems, distributed network systems, robotic manipulator systems and wireless communication systems [1–7]. On the hypothesis of known transition probabilities (TPs), fruitful results on stability, stabilization, sliding mode control, $H_2$ and $H_\infty$ control are reported in [1–5] and the references therein. However, there exist some limitations for the obtained results to practical engineering problems because it is difficult to measure or estimate all TPs. To shorten the gap between theory and practical applications, some results on MJSs with general TPs are carried out in [6–18]. To mention a few, $\mathcal{L}_2 - \mathcal{L}_\infty$ filtering of neutral Markov switching systems with partially unknown TPs is presented in [8]. Finite-time $H_\infty$ control of singular MJSs with partly unknown transition rates is discussed in [11]. C. Morais [13] derives the $H_\infty$ control of polytopic continuous-time MJSs with uncertain TPs.

Alternatively, time-varying delay appears in many systems, which often degrades the system performance or even causes the instability [19–40]. Hence, the stability analysis of MJSs with time-varying delay has attracted much interest [19,20,30–33]. To obtain delay-dependent criterion via the Lyapunov–Krasovskii functional (LKF) method, a challenging problem is how to cope with the integral term $\int_a^b \dot{x}^T(s)R\dot{x}(s)ds$. Around this problem, the descriptor model transformation technique [11,21,23], together with Park or Moon inequality [14,22,27,29], is applied to handle the integral term. However, extra

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dynamics and conservativeness may be caused by the model transformation. To render less conservative results, the free-weighting-matrix (FWM) approach is proposed by [11,14,24,25,28,36]. Unfortunately, introduced slack variables bring heavy computation complexity. Recently, the Wirtinger-based inequality technique could give less conservativeness and slack matrices [26,30,38,39]. Among the above delay-dependent results[36–40], time-delays are assumed to be constant or varying between zero and an upper bound. To date and the best of our knowledge, $H_\infty$ control for continues-time MJSs with interval time-varying delay and incomplete TPs have not been fully investigated in the literature yet, which motivates us to carry out the present work.

In this paper, we further consider the delay-dependent $H_\infty$ state feedback control of MJSs with incomplete TPs. The nonlinearity induced by uncertain and unknown TPs is linearized in the light of the property of TP matrix. Wirtinger-based integral inequality combined with reciprocally convex technique is utilized to tackle the time-varying delay. The coupling among controller gains and system matrices is removed by a constructive method. To make the closed-loop system be stochastically stable with a prescribed $H_\infty$ performance index, sufficient conditions are established by means of linear matrix inequalities. Finally, numerical examples are provided to demonstrate the validity of the established results.

The organization of this paper is as follows. The problem statement and some preliminaries are given in Section 2. $H_\infty$ state-feedback controllers with complete known TPs and incomplete TPs are provided in Section 3, respectively. In Section 4, numerical examples are given to show the effectiveness of the proposed methods. Conclusion is given in Section 5.

**Notation :** $\mathbb{R}^n$ denotes the $n$-dimensional Euclidean space and $\mathbb{R}^{n\times m}$ denotes $n \times m$ real matrices. The notation $R > 0(< 0)$ stands for $R$ is symmetric and positive (negative) definite. $(\cdot)^T$ indicates the transpose of a vector or matrix $(\cdot)$. $*$ represents the symmetry. $0_{n\times m}$ and $I_{n\times n}$ are used to denote the zero block matrix and identity block matrix with compatible dimensions, respectively. $E\{\cdot\}$ means the mathematical expectation operator. For any square matrices $A$ and $B$, define

$$diag\{A, B\} = \begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix}. \quad He(M) = M^T + M.$$

## 2. Problem statement and preliminaries

Consider the following continuous-time MJSs described as

$$\begin{aligned} \dot{x}(t) &= A(r_t)x(t) + A_d(r_t)x(t-d(t)) + B(r_t)u(t) + E(r_t)\omega(t) \\ z(t) &= C(r_t)x(t) + D(r_t)u(t) + F(r_t)\omega(t) \\ x(t) &= \psi(t), \quad t \in [-h_2, 0], \quad r(0) = r_0, \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector of the system; $u(t) \in \mathbb{R}^m$ is control input; $z(t) \in \mathbb{R}^p$ is control output; $\omega(t) \in \mathbb{R}^q$ is the noise signal which is assumed to be an arbitrary signal; $d(t)$ is a time-varying delay satisfying $0 \le h_1 \le d(t) \le h_2$ and $\dot{d}(t) \le \mu < \infty$, meanwhile define $h_{12} = h_2 - h_1$. $\psi(t)$ is vector-valued initial continuous function and belongs to $[-h_2, 0]$. $A(r_t)$, $A_d(r_t)$, $B(r_t)$, $E(r_t)$, $C(r_t)$, $D(r_t)$, and $F(r_t)$ are system matrices. $r_t$ is a continuous Markov process and takes values in $\mathcal{I} = \{1, 2, \ldots, s\}$ and satisfies

$$Pr\{r_{t+h} = j | r_t = i\} = \begin{cases} \pi_{ij}h + o(h), & i \neq j \\ 1 + \pi_{ii}h + o(h), & i = j \end{cases} \tag{2}$$

where $h > 0$, $\pi_{ij} \ge 0$ for $i \neq j$ and $\pi_{ii} = -\sum_{j=1, j\neq i}^{s} \pi_{ij}$ for each mode $i$, $\lim_{h\to 0} o(h)/h = 0$. As a sequence, the corresponding transition probability matrix is

$$\begin{bmatrix} \pi_{11} & \pi_{12} & \cdots & \pi_{1N} \\ \pi_{21} & \pi_{22} & \cdots & \pi_{2N} \\ \vdots & & & \\ \pi_{N1} & \pi_{N2} & \cdots & \pi_{NN} \end{bmatrix}.$$

However, it is hard and costly to measure TPs exactly. Like [5–7], TPs in this paper are assumed to be known, uncertain with known lower and upper bounds and completely unknown. To see the incomplete TPs clearly, the TP matrix with four modes is given below:

$$\begin{bmatrix} \pi_{11} & ? & \pi_{13} & \pi_{14} \\ ? & \pi_{22} & ? & \pi_{24} \\ \alpha_{31} & ? & \pi_{33} & ? \\ ? & ? & \alpha_{43} & ? \end{bmatrix},$$

where $\alpha_{ij}$ $(\underline{\alpha}_{ij} \le \alpha_{ij} \le \bar{\alpha}_{ij})$ and '?' represent the completely unknown TPs, respectively. To simplify the later presentation, the following sets are employed to all possible cases of TPs

$$\begin{cases} R_k = \{j | \pi_{ij} \ is \ known\}, \\ R_{uk1} = \{j | \pi_{ij} \ is \ uncertain \ with \ known \ lower \ and \ upper \ bounds\}, \\ R_{uk2} = \{j | \pi_{ij} \ is \ completely \ unknown\}. \end{cases}$$