Applied Mathematics and Computation 301 (2017) 193–200

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Neural network implementation of inference on binary Markov random fields with probability coding

Zhaofei Yu[a], Feng Chen[a,b,*], Jianwu Dong[a]

[a] Department of Automation, Center for Brain-Inspired Computing Research, Tsinghua University, Beijing 100084, China
[b] Beijing Key Laboratory of Security in Big Data Processing and Application, Beijing 100084, China

ARTICLE INFO

Keywords:
Markov random fields
Approximate inference
Neural network implementation
Hopfield network
Probability coding

ABSTRACT

Markov random fields (MRF) underpin the solution to many problems in computational neuroscience. However, how the inference for MRF could be implemented with neural network is still an important open question. In this paper, we build the relationship between inference equation of MRF and the dynamic equation of the Hopfield network with probability coding. We prove that the membrane potential in the Hopfield network varying with respect to time can implement marginal probabilities inference on binary MRF. Theoretical analysis and experimental results show that our neural network can get comparable results as that of loopy belief propagation (LBP).

1. Introduction

Markov random fields (MRF) are a kind of undirected probabilistic graphical models [1], which have been widely used in various fields, such as computer vision [2,3], magnetic resonance imaging [4,5] and computational neuroscience [6,7]. In neuroscience, MRF have been proved to be extremely useful in explaining some physiological and psychological experiments [6,7]. Despite this, an important open question is how the inference for MRF could be implemented with neural network or neural circuit. Solving this problem is of great importance to both neuroscience and computer science. For one thing, it would help us to understand how human brain performs inference. For another, a brain-like machine could be developed to imitate some of the capabilities of human brain.

Recently, researchers attempt to build the relationship between inference equation of probabilistic graphical models and dynamic equation of neural network and neural circuits. Rao [8,9] proves that the dynamic equation of recurrent neural circuit can implement posterior probabilities inference on a hidden Markov model, under the condition that the firing rate of neuron is proportional to the log of posterior probabilities. Ott and Stoop [7] propose that the Hopfield network is able to implement belief propagation algorithm of binary MRF. However, this method suffers from two shortcomings. Firstly, they do not explain what the dynamics of spiking neuron in Hopfield networks represents. Secondly, the neurons in their network must have specially initialized messages, which is implausible for human brain [7].

In this paper, we build the relationship between inference equation of MRF and the dynamic equation of the Hopfield network with probability coding. We show that the membrane potential of neuron in the Hopfield network represents the difference between the probabilities for two states. What is more, we prove that the membrane potential in the Hopfield network varying with respect to time can also implement marginal probabilities inference on binary MRF. The neural

* Corresponding author at: Department of Automation, Center for Brain-Inspired Computing Research, Tsinghua University, Beijing 100084, China.
E-mail address: chenfeng@mail.tsinghua.edu.cn (F. Chen).

http://dx.doi.org/10.1016/j.amc.2016.12.025

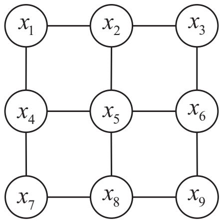

Fig. 1. A pairwise Markov network.

network does not need specially initialized messages when performing inference. Experiments demonstrate that recurrent neural circuits can get comparable results as that of loopy belief propagation (LBP).

2. Preliminaries

In this section, we briefly review Markov random field and an approximate inference method named LBP.

2.1. Markov random field

MRF are a kind of undirected probabilistic graphical model and a joint distribution $P(x_1, x_2, \dots, x_n)$ is defined on the graph [1]. The joint distribution can be factorized into a product of potential functions according to the graph [10]. As illustrated in Fig. 1, for this MRF, $P(x_1, x_2, \dots, x_n)$ has the form:

$$P(x_1, x_2, \dots, x_n) = \frac{1}{Z} \prod_{(i,j)\in E} \psi_{ij}\left(x_i, x_j\right) \prod_{i\in V} \psi_i(x_i), \tag{1}$$

where V represents the set of nodes and E represents the set of edges. $\psi_i(x_i)$ and $\psi_{ij}(x_i, x_j)$ are the unary and pairwise potential functions, respectively. Note that for high-order MRF, potential functions can be defined on more than two variables. Z is the partition function, which is a normalized constant, and $Z = \sum_{x_1, x_2, \dots x_n} \prod_{(i,j)\in E} \psi_{ij}(x_i, x_j) \prod_{i\in V} \psi_i(x_i)$. Eq. (1) also can be reformulated as:

$$P(x_1, x_2, \dots x_n) \ = \frac{1}{Z} \exp\left(\sum_{(i,j)\in E} \theta_{ij}\left(x_i, x_j\right) + \sum_{i\in V} \theta_i(x_i) \right), \tag{2}$$

where $\theta_{ij}(x_i, x_j) = \ln \psi_{ij}(x_i, x_j)$ and $\theta_i(x_i) = \ln \psi_i(x_i)$. Similar to [7], we assume that $\theta_{ij}(x_i, x_j) = \theta_{ij} x_i x_j$ and $\theta_i(x_i) = \theta_i x_i$, where θ_{ij} and θ_i are constants.

2.2. Loopy belief propagation algorithm

The inference problem of MRF is to calculate the marginal probability $P(x_i)$, which is given by:

$$P(x_i) = \sum_{\mathbf{x}/x_i} P(x_1, x_2, \dots x_n). \tag{3}$$

The above equation can be computed directly with variable elimination. However, it has been proved that exact inference of MRF is an NP-complete problem [1,11]. An efficient approximate inference algorithm is LBP, which has been indicated to be a probable way to perform inference for human brain by physiological and psychological experiments [8,12,13]. The main principles of LBP lie in converting the inference problem to the problem of minimizing the Bethe free energy [14,15], which is then solved by an iterative optimization algorithm. In each iteration, every node updates its beliefs (marginal probability) according to the information of all the neighboring nodes. These informations are called message. To be specific, the message in the $t+1$ iteration is [15]:

$$m_{i\to j}^{t+1}\left(x_j\right) \propto \sum_{x_i} \psi_i(x_i) \psi_{ij}\left(x_i, x_j\right) \prod_{k\in N(i)\setminus j} m_{k\to i}^{t}(x_i), \tag{4}$$

where $m_{i\to j}^{t+1}(x_j)$ represents the message sent from node i to node j in the $t+1$ iteration. Note that the message is related to the state x_j. $N(i)\backslash j$ represents all neighboring nodes of node i expect node j. We use $\propto$ to normalize the message and keep $\sum_{x_j} m_{i\to j}^{t+1}(x_j) = 1$. When all the messages converge to the fixed points, the approximate marginal probability $P(x_i)$ is:

$$P_i(x_i) \propto \psi_i(x_i) \prod_{j\in N(i)} m_{j\to i}^{\infty}(x_i). \tag{5}$$