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Harmonic waves in anisotropic nematic elastomers

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ABSTRACT

The problem of plane waves propagation in the nematic elastomers has been investigated. The phase velocity corresponding to primary (P) and secondary waves (S) in the anisotropic nematic elastomers are complex and depend on their angles of propagation. The phase velocity and the attenuation coefficients for these waves are obtained analytically and numerically for a particular model.

1. Introduction

Nematic elastomers are materials of simultaneous combination of elastic properties of rubbers with anisotropy of liquid crystals. Such materials consist of the networks of elastic solid chains formed by the cross linking of nematic crystalline molecules called mesogens as the elements of their main-chains and pendant side-groups. The stress on the polymer network influences the nematic order and the changes in the orientational order affect the mechanical shape of the elastomer. Interplay between elastic and orientational changes is responsible for many fascinating properties of such materials which make them different from the classical elastic solids and liquid crystals. Liquid crystalline elastomers (LCE) [1–5] has many applications in the fields of mechanical actuators (artificial muscles), optics and coatings of materials, which can dissipate mechanical energy.

The electroclinic (EC) effect of an electro-optical coupling is observed in liquid crystals, which consists in the rotation of the optical axis about an electric field, perpendicular to the optical axis itself. The tilt is linear in the electric field and the EC coefficient is a property of the material. Greco and Ferrarini [6] derived the molecular expressions for the EC coefficient and a computational methodology that had allowed its calculations on the basis of the molecular structure. Finkelmann et al. [7] synthesized side chain nematic polymer networks, performed differential scanning calorimetry (DSC), X-ray, birefringence and thermo-mechanical characterizations, and obtained the linear moduli from stress-strain measurements. Selinger et al. [8] discussed a theory for the isotropic-nematic transition in liquid-crystalline elastomers through a variation on Landau theory. DeSimone and Dolzmann [9] analyzed the soft deformation paths and domain patterns in nematic elastomers through the minimization of a nonconvex free-energy. Anderson et al. [10] studied a continuum theory for the mechanical behavior of rubber materials. Teixeira and Warner [11] investigated and discussed analytically and numerically the dynamics of how a nematic elastomer with an anisotropic rubber responds elastically and orientationally to an imposed strain. They obtained different modes decay with either two distinct or with the same exponential laws depending respectively, on whether there is or there is not complete reorientation of the molecular long axes. Nematic elastomers [12] exhibited the remarkable phenomenon of soft or semisoft elasticity in which the effective shear modulus for shears in planes containing the anisotropic axis, respectively, vanishes or is very small. Problems of nematic elastomers are also available in Uchida

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[13], Selinger et al. [14], DeSimone and Teresi [15], Ericksen [16,17], Leslie [18,19], Weilepp and Brand [20], Paolo Cermelli et al. [21], Agrawal et al. [22], monograph of Warner and Terentjev [23], Cmok et al. [24] and Petelin and Copic [25].

The problem of waves and vibration is an important area of research. Wu et al. [26] attempted the problem of the analysis of phase velocity and polarization features for elastic waves in tilted transverse isotropy (*TTI*) media. They derived the approximate phase velocity for *qP*, *qSV* and *qSH* waves in such media based on the Thomson dimensionless anisotropic parameters and weak anisotropy approximation theory. Singh and Zorammuana [27] studied the problem of incident longitudinal wave at a fiber-reinforced thermoelastic half-space and obtained the reflection and energy ratios of reflected elastic waves. Singh and Bijarnia [28] investigated the problem of the propagation of plane waves in anisotropic two temperature generalized thermoelasticity. Yang et al. [29] discussed the problem of Rayleigh wave propagation in nematic elastomers and used the viscoelastic theory to find the dispersion and attenuation properties of the Rayleigh waves. Singh and Singh [30] explained the effect of corrugation on incident qSV-waves in pre-stressed elastic half-spaces thereby obtaining the reflection and transmission coefficients. Singh [31] also obtained the reflection and transmission coefficients of the reflected and transmitted waves from a plane interface between two dissimilar half-spaces of thermo-elastic materials with void. Fradkin et al. [32] developed the spectral and polarization properties of acoustic waves propagating in nematic liquid-crystalline rubber materials. The perturbation of phase speed and attenuation of the waves for the problems of nematic coating has been studied by Zakharov [33,34] and Zakharov and Kaptsov [35]. Singh [36] obtained the reflection coefficients of the incident *qP* and *qSV*-waves from a free surface of nematic elastomer half-space using appropriate boundary conditions. Terentjev et al. [37] developed a theory of elastic waves in oriented monodomain nematic elastomers using the Leslie–Ericksen theory of anisotropic viscous dissipation in a nematic liquid.

In this paper, we discussed the problem of harmonic waves in the anisotropic nematic elastomer and obtained their phase velocity. These phase velocities are complex and depend on their angles of propagation. The phase velocity and their attenuation coefficients of these waves are computed numerically for a particular elastomer.

2. Basic equations

The elastic potential energy density in nematic solid takes the form [37] as

$$F = C_1(\mathbf{n}\cdot\epsilon\cdot\mathbf{n})^2 + 2C_2 tr[\mathbf{e}](\mathbf{n}\cdot\epsilon\cdot\mathbf{n}) + C_3(tr[\mathbf{e}])^2 + 2C_4(\mathbf{n}\times\epsilon\times\mathbf{n})^2 + 4C_5(\mathbf{n}\times(\epsilon\cdot\mathbf{n}))^2 + \frac{D_1}{2}(\mathbf{n}\times\mathbf{\Theta})^2 + D_2\mathbf{n}\cdot\epsilon\cdot(\mathbf{n}\times\mathbf{\Theta}), \tag{1}$$

where the Frank elastic energy describes the non-uniform directors is not included due to the assumption of uniform director rotations in nematic elastomers, $\mathbf{\Theta} = \mathbf{\Omega} - (\mathbf{n}\times\delta\mathbf{n})$ is an independent rotational variable, $\delta\mathbf{n}$ is a small variation in the undistorted nematic director, $\mathbf{n}\cdot\mathbf{\Omega} = (1/2)\mathrm{curl}\,\mathbf{u}$ is the local rotation vector, $(\mathbf{n}\times\delta\mathbf{n})$ are director rotations, $\epsilon_{ij} = e_{ij} - (1/3)tr[\mathbf{e}]\delta_{ij},\ (i,j=1,2,3)$ is the traceless part of linear symmetric strain, $e_{ij} = (1/2)(\delta_j u_i + \delta_i u_j)$, C_i are elastic constants and D_1, D_2 are coupling constants.

Using the Leslie–Ericksen theory [16–19] of anisotropic viscous dissipation in nematic liquid, the Rayleigh dissipation function (entropy production density) can be written in the quadratic form of corresponding velocities [32] as

$$T\dot{s} = A_1(\mathbf{n}\cdot\dot{\epsilon}\cdot\mathbf{n})^2 + 2A_2 tr[\dot{\mathbf{e}}](\mathbf{n}\cdot\dot{\epsilon}\cdot\mathbf{n}) + A_3(tr[\dot{\mathbf{e}}])^2 + 2A_4(\mathbf{n}\times\dot{\epsilon}\times\mathbf{n})^2 + 4A_5[\mathbf{n}\times(\dot{\epsilon}\cdot\mathbf{n})]^2 + \frac{1}{2}\gamma_1(\mathbf{n}\times\dot{\mathbf{\Theta}})^2 + \gamma_2\mathbf{n}\cdot\dot{\epsilon}\cdot(\mathbf{n}\times\dot{\mathbf{\Theta}}), \tag{2}$$

where A_i are viscous coefficients. This equation describes two types of dissipation, by shear flow and by rotation of the director, and vanishes for rigid rotations.

The equations of motion of viscous nematic solid in the absence of the effects of Frank elasticity on the director gradient are given as [32]

$$\nabla\cdot\sigma = \rho\ddot{\mathbf{u}}, \tag{3}$$

$$0 = \mathbf{n}\times[(D_1+\gamma_1\delta_t)\mathbf{n}\times\mathbf{\Theta} + (D_2+\gamma_2\delta_t)\mathbf{n}\cdot\epsilon], \tag{4}$$

where $\mathbf{u} = (u_1, u_2, u_3)$ and the second equation is the balance of torques.

The components of the viscoelastic symmetric stress tensor with the choice of the coordinate x_3-axis to lie in the direction of the undistorted director, $\mathbf{n}$ are

$$\tau_{11} = (1+\tau_R\partial_t)(c_{11}\epsilon_{11} + c_{12}\epsilon_{22} + c_{13}\epsilon_{33}),$$

$$\tau_{22} = (1+\tau_R\partial_t)(c_{12}\epsilon_{11} + c_{11}\epsilon_{22} + c_{13}\epsilon_{33}),$$

$$\tau_{33} = (1+\tau_R\partial_t)(c_{13}\epsilon_{11} + c_{13}\epsilon_{22} + c_{33}\epsilon_{33}),$$

$$\tau_{12} = \tau_{21} = 2(1+\tau_R\partial_t)c_{66}\epsilon_{12},$$

$$\tau_{13} = 2(1+\tau_R\partial_t)c_{44}\epsilon_{13} - \frac{1}{2}D_1(1+\tau_2\partial_t)\Theta_2,$$