Applied Mathematics and Computation 302 (2017) 122–143

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Modeling the dynamics of stage-structure predator-prey system with Monod–Haldane type response function

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ARTICLE INFO

Keywords:
Monod–Haldane functional response
Predator-prey model
Stage-structure
Hopf-bifurcation

ABSTRACT

A stage-structure predator prey model is proposed and analyzed in this paper in which predators are divided into juvenile and mature predators using Monod–Haldane-type response function. The dynamical behavior of this system both analytically and numerically is investigated from the view point of stability and bifurcation. We investigate global stability around the interior equilibrium point E^* by constructing suitable Lyapunov function. Our model simulation indicates that the conversion of prey population to juvenile predators can destabilize the model system which lead to limit cycle oscillations. We also investigate that the rate of juvenile predators becoming mature predators play an important role to destabilize the model system for the stable coexistence of both the populations. We carried out extensive numerical simulations of the model to confirm the analytical findings.

1. Introduction

Predator-prey dynamics is one of the dominant field in theoretical ecology as well as in applied mathematics which has been studied by many researchers over the last few decades, and mathematical models have been used to better understand these complex phenomena. While going for the relationship between predator-prey interaction through mathematical model, it seems to be very simple initially but later it becomes challenging problems from the perspective of applied mathematicians. The most harder and crucial phase of modeling the predator-prey ecosystem is to examine and validate whether the proposed mathematical model can demonstrate the proper behavior for the system under consideration.

The well-known continuous population models such as logistic model and Lotka–Volterra models overlook stage-structures and size structures. But, in many cases, population depends on size-structure, stage-structure or developmental stage, and these may influence the outcomes of population evolutions. The inclusion of size-structure, stage-structures in a population model is one of the typical ways to introduce life history.

While we go through the review of the research work on the predator-prey dynamics in addition of stage-structure for the predators, we have seen that there are several works on the dynamics of predators and their functional responses. Thieme [20] and Kostova [19] studied a population model through a system of ordinary differential equations which divide into mature and immature species and mathematically they showed that all trajectories of the model system converges to an equilibrium point if the transition from immature to mature stage is influenced by the intra-immature competition in a weaker way only. Wang and Chen [21] investigate the interactions between predator and prey species by incorporating stage-structure for the predators via a system of ordinary differential equations and they observed that their

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http://dx.doi.org/10.1016/j.amc.2017.01.019
0096-3003/

model admits oscillatory behavior. Khajanchi [22] investigate a mathematical model by introducing stage-structure on predators and the model exhibits Hopf bifurcation and a stable limit cycle appear at the bifurcation point. Thus, the heterogeneity of juvenile and adult predators is one possible factor for periodic oscillations of the predators and prey. Gourley and Kuang [23] discussed the effect of constant maturation time delay in a stage structure predator-prey interaction model in which the discrete time delay able to generate sustainable oscillatory dynamics. Aiello et al. [24,25] investigate the effect of discrete time lag in a single species population model, where the population divide into juvenile and adult predators. Their model demonstrate global attractor but the stage-structure does not exhibit oscillatory behavior in a case of single species population.

A large body of works have been done by numerous researchers on the predator-prey interactions by incorporating stage-structures [4,5,17–25,31,32]. However, in most of the works researchers presume that the transition rate from juvenile to adult stage is constant. This provides that adult individuals have sufficient food to survive at any time. This might be the case where juvenile predators feed on a resource that is different from prey and is very rich. If the food of juvenile predators is obtained from their parents (mature predators) or is obtained by themselves via attacking the prey, the transition rate is directly influenced by the available nutrient, which may provide rich dynamical scenarios of predator-prey relationships. A robust prey-dependent consumption prey-predator interaction with Gompertzian growth model and periodic harvesting for the prey and stage-structure for the predator species with constant maturation time delay has been studied by Liu et al. [5]. Kar [3] studied a predator-prey model through a system of ordinary differential equations by incorporating stage-structure for the predators and discussed the harvesting strategy by outside control. Bandopadhayay and Banerjee [4] used a mathematical model to study the stage-structure predator-prey system in a delayed version where they showed that their model undergoes Hopf bifurcation when the discrete time delay τ crosses the threshold value τ_0 and also they calculate the length of gestation delay.

In theoretical ecology, there are several well-known response functions in the predator-prey interactions which refer to as Holling type-I (linear), type-II (cyrtoid), type-III (sigmoid), type-IV, Beddington–DeAngelis-type response function and Hassel–Verley type response function etc. Researchers around the world discussed and raised some open questions regarding stage-structure predator-prey relationships with various kind of response functions. Holling type-IV response function is relatively less investigated in the theoretical ecology. In 1930, it was first introduced by Haldane [26] in enzymology. Later, Sokol and Howell [16] derived a simplified Monod–Haldane response function, utility and significance of such response function is derived in the next section. Few literatures used modified Monod–Haldane-type response functions [27–30] to describe the relationship between predator-prey system. Influenced by those literatures we have chosen the modified Monod–Haldane response function to study the stage-structure predator-prey system with stage-structure for the predators.

Our objective of this paper is to investigate a three-tier continuous model of prey-juvenile predator-adult predators with stage structure for the predators via a Monod–Haldane-type response function to model the parental (adult predators) care for the juvenile predators. Here, we are interested to investigate the change in dynamical behavior of juvenile and adult predators with the interaction of prey species when there is a transition from prey to juvenile predators and the conversion rate (α) of immature predators to mature predators influence the dynamics of the predator-prey system. The transition from prey to juvenile predators is incorporated into our modeling approach via Monod–Haldane-type response function, advantage and importance for the choice of such response function is discussed in the next section. We consider that mature predators forage for the prey and provide parental care to their offsprings. In contrast, the paper by de Ross and Persson [31,32] discussed the model of a resource and a consumer individuals with exploitative competition by immature and mature predators.

In this article, we present a mathematical model depicting the dynamical behavior of stage-structured predator-prey interactions via Monod–Haldane-type response function. The aim is to capture the dynamics of the model due to incorporation of stage-structured on predators. In Section 2, we present the formulation of the mathematical model through a system of nonlinear ordinary differential equations and the positivity and boundedness of its solution. Qualitative analysis of the model system is performed in Section 3, which includes the number of feasible equilibria and their stability analysis. Global stability analysis about the interior equilibrium point as well as Kolmogorov analysis of the two dimensional subsystems of the original three dimensional system is performed in Section 4. We also investigate the conditions for the existence of Hopf bifurcation around the interior steady state E^* along with its direction and stability. In Section 5, we numerically illustrate our results and obtain very rich dynamics of the model under consideration. A final discussion concludes the paper for both the predators and the prey population.

2. Mathematical model

In population dynamics, a functional response is the rate at which predator species capture prey individuals. In the dynamics of microbial or chemical kinetics, the functional response designates the uptake of substrate by the micro-organisms. The simplest form of functional response is obtained by presuming that in the time available for searching, the total change of prey density is proportional to prey density. Hence, if $x(t)$ designates the prey individuals at time t, then its functional response $\alpha x(t)$, with $\alpha > 0$. In 1925, Lotka and in 1926, Volterra studied such type of functional response in studying a hypothetical chemical reaction. However, the graph plotted by Lotka–Volterra response function is a straight line passing through origin and is unbounded, which is unrealistic. Thus, the response functions need to be bounded and nonlinear/non-