



# Selection intensity and risk-dominant strategy: A two-strategy stochastic evolutionary game dynamics in finite population



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## ABSTRACT

Stochastic evolutionary game dynamics with weak selection in finite population has been studied and it has been used to explain the emergence of cooperation. In this paper, following the previous studies, the diffusion approximation of a two-strategy stochastic evolutionary game dynamics in finite population that includes a small mutation rate between two strategies is investigated, where we assume that these two strategies are both strict Nash equilibrium (NE). Our main goal is to partially reveal the effect of selection intensity on the stochastic evolutionary game dynamics. Through the analysis of potential function of the stationary distribution, our main result shows that for all possible situations with that the selection intensity is not zero (that includes the strong selection), if a strategy is a risk-dominant NE, then its expected fitness with respect to the stationary distribution must be larger than that of other strategy. This result not only extends the previous results but also provides some useful insights for understanding the significance of selection intensity in stochastic evolutionary game dynamics in finite population.

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## 1. Introduction

Evolutionary game theory has been successfully applied since its introduction by Maynard Smith and Price [1]. Recently, the stochastic evolutionary game dynamics in the finite population was investigated [2,3], and it has been used to explain the evolutionary emergence of cooperation. Nowak et al. [2] studied a stochastic process for games with two (pure) strategies, denoted by  $A$  and  $B$ , respectively, when population size is fixed at  $N$ . They introduced the concept of one-third law to show why the selection could favor a single mutant using strategy  $A$  replacing a resident population using the other pure strategy  $B$ . In this basic form, the one-third law connects the Nash equilibrium (NE) concepts of this two-strategy game to stochastic properties of the birth-death Markov system known as the Moran process [2–4]. Specifically, Nowak et al. [2] showed that, when  $A$  and  $B$  are both strict NE, population size is large and the selection is weak, the probability that this replacement occurs under the Moran process is higher than that obtained from neutral drift if the completely mixed NE of this game plays strategy  $A$  less than one-third of the time (see also [3,5]). Taylor et al. [6] investigated the evolutionary significance of one-third law in the Moran process (see also [7]).

The diffusion approximation of the stochastic evolutionary game dynamics based on the Moran process was first studied by Traulsen et al. [8,9]. For the situation with weak selection, their main results exactly match the Nowak et al.'s results [2]. Zheng et al. [10] also used the diffusion approximation of stochastic evolutionary game dynamics to reveal the evolutionary

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significance of the one-third law, and they showed that the one-third law implies that the difference between the average fitness of strategies  $A$  and  $B$  is positive when a single individual using  $A$  is initially present in a large population that consists otherwise of individuals using  $B$ . On the other hand, Zheng et al. [10] connected the risk-dominance of strategy  $A$  to its larger expected fitness with respect to the stationary distribution of the diffusion approximation that includes a small mutation rate between the two strategies. All of these results are based on the assumption that the selection is weak. However, Traulsen et al. considered stochastic evolutionary game dynamics for individual and group selection at any intensity [11,12]. In this paper, to show the effect of selection intensity on the stochastic evolutionary game dynamics, or the relationship between the risk-dominance, expected fitness with respect to the stationary distribution and selection intensity, we will study how the selection intensity influences the characteristics of potential function of the stationary distribution. Our results may provide some useful insights into understanding the significance of selection intensity in stochastic evolutionary game dynamics in finite population.

We have also noticed that the evolutionary graph theory and spatial games have been developed to reveal the evolution of cooperation in structured population [3]. Some studies on the stochastic game dynamics in structured population suggested that the selection mechanism for promoting cooperation in structured population could be significantly different from that of unstructured population [13–18], and some researchers also considered the effect of selection intensity on the evolutionary graph theory [19]. Although in this study we consider only the situation with well-mixed population, we will extend our analysis to the situation with structured population in the next step.

## 2. Model and analysis

Similar to Traulsen and Hauert [7] (see also [2,5,10]), we consider also a simple two-strategy stochastic evolutionary game dynamics in a finite population, where the payoff matrix is given by

$$\begin{array}{c} A \\ B \end{array} \begin{array}{cc} A & B \\ \left( \begin{array}{cc} a & b \\ c & d \end{array} \right) \end{array}, \quad (1)$$

where the payoff of an individual displaying strategy  $A$  is  $a$  ( $b$ ) when it plays against an individual displaying strategy  $A$  ( $B$ ) and the payoff of an individual displaying strategy  $B$  is  $c$  ( $d$ ) when it plays against an individual displaying strategy  $A$  ( $B$ ). Without loss of generality, we here assume that all the entries of the payoff matrix are positive. The population size,  $N$ , is assumed to be fixed, the number of individuals displaying is denoted by  $j$  and the number of individuals displaying  $B$  is  $N-j$ . For the frequency-dependent Moran process, the expected payoffs of  $A$  and  $B$  in random pairwise interactions, denoted by  $f_j$  and  $g_j$ , respectively, are

$$\begin{aligned} f_j &= (1 - \omega) + \omega \frac{a(j-1) + b(N-j)}{N-1}, \\ g_j &= (1 - \omega) + \omega \frac{cj + d(N-j-1)}{N-1}, \end{aligned} \quad (2)$$

where the parameter  $\omega$  denotes the selection intensity with  $0 \leq \omega \leq 1$ , which represents the contribution of the game to individual fitness [2,3].

We here still assume that, at each time step, an individual is chosen for reproduction proportional to its fitness, and that the one identical offspring produced replaces another randomly chosen individual [2]. For this system, selection is said to favor  $A$  replacing  $B$  if, when there is initially one  $A$  individual in the population (or the initial frequency of  $A$  is  $1/N$ ), the fixation probability of  $A$ , denoted by  $\rho_A$ , is larger than  $1/N$ , which is the fixation probability under neutral selection ( $\omega=0$ ). Nowak et al. [2] have shown that, under the Moran process with weak selection (specifically,  $N\omega \ll 1$ ), selection favors  $A$  replacing  $B$  if and only if  $(N-2)a + (2N-1)b > (N+1)c + (2N-4)d$ , and that if both  $A$  and  $B$  are strict NE for the payoff matrix given in Eq. (1) (this means that the payoff matrix satisfies  $a > c$  and  $d > b$ ), then  $\rho_A > 1/N$  if  $x^* < 1/3$ , where  $x^* = (b-d)/(b-d+c-a)$  is the frequency of  $A$  at the completely mixed NE of this game [2]. This is also called the one-third law [3]. Nowak et al. [2] also pointed out that, when both  $A$  and  $B$  are strict NE,  $\rho_A > \rho_B$  (where  $\rho_B$  denotes the fixation probability of strategy  $B$ , the inequality  $\rho_A > \rho_B$  means  $A$  is more likely to replace  $B$  than vice versa) under the Moran process with weak selection and large population size if and only if  $A$  is risk dominant (i.e.  $x^* < 1/2$ ). In this study, we still assume both  $A$  and  $B$  are strict NE.

For large  $N$ , the fitness of  $A$  and  $B$  can be approximated as

$$\begin{aligned} f(x) &\approx (1 - \omega) + \omega(xa + (1-x)b), \\ g(x) &\approx (1 - \omega) + \omega(xc + (1-x)d), \end{aligned} \quad (3)$$

respectively, where  $x$  denotes the frequency of  $A$ ,  $x = j/N$ . Thus, it is easy to see that for the diffusion approximation of this process, the transition probabilities from  $x$  to  $x + 1/N$  and from  $x$  to  $x - 1/N$  can be given by

$$\pi^+(x) = (1-x) \frac{xf(x)(1-\mu) + (1-x)g(x)\mu}{xf(x) + (1-x)g(x)},$$

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