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Fast weighted TV denoising via an edge driven metric

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ABSTRACT

In this paper we propose a new Fast Weighted Total Variation denoising approach, where we introduce edge driven weights in the standard TV discrete regularizer and a non Euclidean metric in the discrepancy term, induced by a positive definite matrix B, strictly related to the weights. In this way the fidelity constraint is adapted according to "edgeness" of each pixel. The corresponding minimization problem is iteratively solved by using the Split-Bregman strategy, in which, due to the particular choice of the structure of the positive definite matrix involved in the measure of the fidelity term, the optimality conditions imposed for the computation of the minimum are reduced to simple assignments, since all variables are decoupled. For its solution we propose a Fast Weighted Total Variation (FWTV) algorithm and, moreover, we prove its convergence. Several experiments demonstrate that the FWTV algorithm outperforms, both in terms of accuracy and execution times, the performance of the Weighted Split-Bregman denoising approach, where the ℓ_2- norm is used in order to measure the fidelity term. In the case of synthetic images, the proposed algorithm is better respect to the best-state-of-art algorithms, but the methods not based on TV minimization give better performances with respect to our proposal in the case of natural images.

1. Introduction

Image denoising plays an important role in various applied areas, such as medical imaging, video processing, object recognition, image segmentation, and so on. The objective of image denoising is to recover the unknown true image u from a noisy measured image $g = u + z$, where z is a random noise term. Due to the inverse nature of this problem, variational approaches have gained a wide popularity in recent years, because of the possible addition of well-chosen regularity terms. A general model for the image denoising is:

$$u^* = \arg\min_u J(u) + \frac{\mu}{2} H(u), \tag{1}$$

where $H(u)$ represents the fidelity term, $J(u)$ the regularization term and μ is a positive balancing parameter. Among the most influential denoising variational approaches, the Total Variation (TV) minimization framework, introduced by Rudin and Osher [1] and Rudin, Osher and Fatemi [2], recovers the denoised image by minimizing the ℓ_2-norm fidelity regularized by the TV of the image. Variants and improvements of this approach have been proposed in the past two decades: the squared ℓ_2-norm in the fidelity term was replaced by the ℓ_1-norm in [3,4]; in [5,6] the authors proposed a nonlocal TV regularization, while in [7] the authors minimized TV with a nonlocal data fidelity term. In [8] and in [9] the authors

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proposed a weighted difference of anisotropic and isotropic TV and an Oriented Total variation $\ell 1/2$ as regularization term, respectively. In [10] the authors presented the spatially adaptive total variation model that applies less smoothing near significant edges by utilizing a spatially varying weight function that is inversely proportional to the magnitude of image derivatives, while in [11] the authors proposed the use of a new edge indicator, named difference curvature, which can effectively distinguish between edges and ramps, in an adaptive TV method. In [12] the authors propose a spatially adapted TV-algorithm (SA-TV), that utilizes a spatially dependent regularization parameter in the context of a multi-scale total variation model, in order to enhance image regions containing details while still sufficiently smoothing homogenous features. In [13] the authors proposed the nonconvex SCAD regularization and the use of a majorization–minimization algorithm in order to overcome the nonconvexity of $J(u)$. In [14] a non-convex hybrid TV regularization method is presented by using iterative reweighted method and a solution is obtained with the augmented Lagrangian based algorithm. In [15] the authors develop algorithms for constrained minimization of the total p-variation (TpV), ℓ_p of the image gradient. In [16] the use of parameterized non-convex regularizers has been proposed in order to effectively induce sparsity of the gradient magnitudes in the solution, while maintaining strict convexity of the total cost functional.

Motivated by the need of preserving sharp edges, while removing noise, we propose a Fast Weighted Total Variation denoising approach (FWTV), which performs an adaptive edge driven regularization. Normally the data fidelity term represents the observation process (e.g. additive noise), while the regularization term represents prior assumption about the image. In this paper, the prior assumption about the image (specifically its edgeness) is used also in the data fidelity term. The euclidean norm in the fidelity term assumes that the image is smooth through-out. Instead, the use of the prior assumption about the edgeness also in the fidelity term makes robust the regularization process. In particular, the pixels belonging to an edge will have a greater fidelity with respect to pixels belonging to uniform area.

Our proposal comprises two new contributions: the first is that the regularization is performed by means of anisotropic weighted total variation, where the weights are evaluated using the derivative of a strongly non-convex function of the image gradient. The second is the use of the above edge driven weights in the definition of a positive definite matrix, whose induced metric is used to measure the fidelity of the reconstruction. The advantages of using this new metric are twofold: the fidelity constraint is adapted according to ”edgeness” of each pixel and the particular choice of the matrix structure leads to a great reduction of the computational effort for the solution of the denoising problem.

The corresponding minimization problem is iteratively solved by using the Split-Bregman strategy. Due to the particular choice of the positive definite matrix involved in the measure of the fidelity term, the optimality conditions imposed for the computation of the minimum are reduced to simple assignments, since all variable are decoupled. We prove the convergence of the proposed FWTV algorithm.

The paper is organized as follows: in Section 2 we briefly review the Weighted Total Variation model based on the Split Bregman strategy, where the ℓ_2-norm is used in the fidelity term. In Section 3 we describe the proposed FWTV algorithm and we prove its convergence. In Section 4 we test its numerical performances in comparison with the best-state-of-art algorithms. Finally, in Section 5 we report our conclusions.

2. Review of the weighted TV model

Over the years, the ROF model for image denoising has been improved by adding several generalizations. In particular we recall the Weighted Total Variation model, where the suitably chosen weights allow us to distinguish between edges and smooth areas in the image.

Let the measured image be

$$g = u + z, \tag{2}$$

where u is the true image and z is the noise term with i.i.d. Gaussian entries of zero mean and variance σ^2.

By denoting u as the vector in $\mathbb{R}^{N^2}$ obtained by a lexicographical ordering of a $N \times N$ 2D image, the anisotropic Weighted TV-regularized model can be cast as [21],[22]

$$u^* = \arg\min_{u \in R^{N^2}} F(u) = \left\{ \frac{\mu}{2} \|u - g\|_2^2 + \|\nabla_x^\alpha u\|_1 + \|\nabla_y^\beta u\|_1 \right\} \tag{3}$$

where $\mu > 0$ is a regularization parameter, and ∇_x^α and ∇_y^β are the vertical and the horizontal weighted difference operators, respectively, mapping $\mathbb{R}^{N^2}$ to $\mathbb{R}^{N^2}$, whose action on a vectorized image is defined as

$$\nabla_x^\alpha u_i = \begin{cases} 0 & \text{for i such that } \bmod(i, N) = 1 \\ \alpha_i (u_i - u_{i-1}) & \text{otherwise} \end{cases} \tag{4}$$

and

$$\nabla_y^\beta u_i = \begin{cases} 0 & \text{for } i = 1, .., N \\ \beta_i (u_i - u_{i-N}) & \text{otherwise} \end{cases} \tag{5}$$

and $\alpha_i > 0$ and $\beta_i > 0$ are constants that weight the first order difference along the vertical and horizontal directions, respectively, and $(\|\nabla_x^\alpha u\|_1 + \|\nabla_y^\beta u\|_1)$ is the anisotropic Weighted TV in the discrete setting. In order to preserve the image