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A stage-structured predator-prey model with predation over juvenile prey

Yang Lu[a], Kasia A. Pawelek[b], Shengqiang Liu[a,*]

[a] Academy of Fundamental and Interdisciplinary Sciences, Harbin Institute of Technology, 3026#, 2 Yi-Kuang Street, Harbin 150080, China
[b] Department of Mathematics and Computational Science, University of South Carolina Beaufort, Bluffton, SC 29909, USA

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ABSTRACT

We formulate and study a stage-structured predator-prey model of Beddington–DeAngelis-type functional response to investigate the impact of predation over the immature prey by the juvenile predator. This kind of predation has been omitted in many of the mathematical models, mainly due to the great challenges in mathematical analysis associated with the complicated exponential and state-dependent prey maturation rate term. We establish the threshold dynamics determined by the net reproductive number of the predator population $\Re_0$. The predator-free equilibrium is globally stable if $\Re_0 < 1$, while the predator persists if $\Re_0 > 1$. Numerical simulations are conducted to illustrate our analytical results. Our results show that an appropriate interference among predators may drive the state of internal coexistence into asymptotic stability.

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1. Introduction

The study of predator-prey interactions has a long history in the field of ecology and mathematical modeling including theoretical, laboratory, and experimental studies [23–25]. Scholarly work has mainly concentrated on predator-one prey systems as a prelude to understanding more complex interactions. Important aspects of the dynamics of predator-one prey, as well as, more complex systems are persistence and stability of equilibria. In this work we obtain survival conditions and final population densities of the predator and prey. Earlier models and approaches mainly focused on the simplified cases, assuming that the predators consume prey of all ages omitting the predator eating preferences and heterogeneity within the prey population [13].

In the natural habitat, the survival of animals often relies on the capture of juvenile prey (or larva-prey), for example, mosquitofish that feed on mosquito larvae [22], and *Cylindrical bark beetles* that feed on the *Asian longhorned beetle* larva [12,21]. Examples of further predator-prey modeling literature include [1,3,5,7–11,22]. In [13] a discrete predator-prey model in which predator consumes only juvenile prey was developed. It was shown that an age-dependent predation can have a stabilizing effect and that the behavior of the model depends critically on the duration of maturation period of the prey, in particular, a very small number of elderly prey represent a powerful stabilizing factor. Additionally, [14] and [15] developed models for the predator-prey interactions with discrete age structure of prey, which lead to changes in stability. A possibility of the stable periodic solutions in a model with a continuous age structure introduced only to the prey population was shown in [16].

* Corresponding author.
E-mail addresses: moonyanglu@126.com (Y. Lu), kpawelek@uscb.edu (K.A. Pawelek), lsqmath@gmail.com, sqliu@hit.edu.cn (S. Liu).

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In [6], Liu and Beretta proposed and studied a structured predator-prey model with the functional response of Beddington–DeAngelis type, by Beddington [2] and DeAngelis et al. [4] independently, for references, see [26,27]. Recently, Gourley and Lou [21] considered a stage-structured predator-prey model with the functional response of Holling type II and obtained rigorous results on the stability of equilibria and threshold dynamics for the persistence and extinction of predators.

Motivated by the above works in [6,21], in this study, we propose and investigate a stage-structured predator-prey model with the functional response of Beddington–DeAngelis type, and similar to [21], we also assume that the immature predator exclusively hunts the juvenile prey. Our work extends the corresponding results in Gourley and Lou [21] by introducing the impact of mutual interference among predators. Comparing with the model [6], the prey maturation term in our model is state-dependent which leads to significant challenges in mathematical analysis. The main aim of our paper is to study the persistence and extinction of the system and to understand survival conditions of the predator and prey.

In the subsequent section we show the derivation of the model. Next, we begin model analysis with proving the positivity and boundedness of solutions and existence of equilibria. In Section 4, we explore sufficient conditions ensuring global stability of the predator-free equilibrium and uniform persistence of the system. We conduct numerical simulations illustrating our analytical results in Section 5. In the last section, we provide a brief discussion and summary of main results.

2. Model derivation

Throughout this article, the subscripts x and y indicate the prey and the predator, respectively. Some variables, such as the per-capita mortality rates, have also superscripts l or a, which stand, for juvenile and adult, respectively. First, we derive an equation for the number $L_x(t)$ of juvenile prey at time t. We introduce $l_x(t, a)$ to denote the density of the immature prey at time t of age a. Immature prey are predated by the juvenile predator. Using a standard von Foerster age-structured modeling approach, we write

$$\frac{\partial l_x(t,a)}{\partial t}+\frac{\partial l_x(t,a)}{\partial a}=-\mu_x^l l_x(t,a)-\frac{e\sigma l_x(t,a)L_y(t)}{1+he\sigma L_x(t)+k_2L_y(t)} \tag{1}$$

to describe the loss of the juvenile prey either due to the natural death with per-capita mortality rate μ_x^l, or through predation by the juvenile predator as described by the last term in (1) using a functional response of Beddington–DeAngelis type with $L_y(t)$ denoting the number of the juvenile predator at time t. This response is further described in (5), in which the corresponding term has $L_x(t)$ in both its numerator and denominator and therefore levels off at large values of $L_x(t)$, thereby modeling an important feature that each the immature predator can consume only a limited quantity of the juvenile prey biomass per unit time. This is an especially relevant observation because a single juvenile prey offers a rather large amount of food to a single immature predator. The coefficient e stands for the prey juvenile biomass encounter rate (the exponential function is denoted by exp throughout, to avoid confusion), k_2 is the magnitude of interference among predators, h is the handling (digestion) time per unit biomass consumed, and σ is the fraction of consumed the prey juvenile biomass. The total number of the juvenile prey at time t is

$$L_x(t)=\int_0^{\tau_x} l_x(t,a)da,$$

where τ_x is the maturation time for the prey. In fact, the juvenile prey do not pupate until they have reached a critical mass, and the time taken to do so depends on the natural factors. Due to the challenges associated with modeling this dependence, we assume that τ_x is a known constant here. Differentiating, and using (1), we have

$$\frac{dL_x(t)}{dt}=l_x(t,0)-l_x(t,\tau_x)-\mu_x^l L_x(t)-\frac{e\sigma L_x(t)L_y(t)}{1+he\sigma L_x(t)+k_2L_y(t)}. \tag{2}$$

Now, $l_x(t, 0)$ is the egg laying rate of the prey; this is taken to be a function $b(\cdot)$ of the total number $A_x(t)$ of adult prey based on the assumption that the adult prey remain reproductively active throughout the majority of their relatively short adult lives. This is thought to be a reasonable assumption for modeling insect populations. Thus,

$$l_x(t,0)=b(A_x(t)). \tag{3}$$

Next, we calculate $l_x(t, \tau_x)$. To do so, we define $l_x^\xi(a)=l_x(a+\xi,a)$. Differentiating, and using (1), we get

$$\frac{dl_x^\xi(a)}{da}=-\mu_x^l l_x^\xi(a)-\frac{e\sigma l_x^\xi(a)L_y(a+\xi)}{1+he\sigma L_x(a+\xi)+k_2L_y(a+\xi)}$$

so that

$$l_x^\xi(a)=l_x^\xi(0)\exp\left[-\int_0^a\left\{\mu_x^l+\frac{e\sigma L_y(\eta+\xi)}{1+he\sigma L_x(\eta+\xi)+k_2L_y(\eta+\xi)}\right\}d\eta\right].$$

Setting $a=\tau_x$ and $\xi=t-\tau_x$ and using (3), we obtain

$$l_x(t,\tau_x)=b(A_x(t-\tau_x))\exp\left[-\int_0^{\tau_x}\{\mu_x^l+\frac{e\sigma L_y(\eta+t-\tau_x)}{1+he\sigma L_x(\eta+t-\tau_x)+k_2L_y(\eta+t-\tau_x)}\}d\eta\right]. \tag{4}$$

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