Applied Mathematics and Computation 298 (2017) 1–12

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

On the solitary wave solution of fractional Kudryashov–Sinelshchikov equation describing nonlinear wave processes in a liquid containing gas bubbles

A.K. Gupta, S. Saha Ray*

National Institute of Technology, Department of Mathematics, Rourkela 769008, India

ARTICLE INFO

MSC:
26A33
35G25
35R11
35Q35
42C40

PAC code:
02.30.Jr

Keywords:
Fractional Kudryashov–Sinelshchikov equation
Radial basis function method
Sech-tanh method
Caputo fractional derivative

ABSTRACT

In the present paper, the nonlinear time-fractional Kudryashov–Sinelshchikov equation has been solved numerically by using radial basis function (RBF) method. The RBF method is successfully implemented to the time-fractional Kudryashov–Sinelshchikov equation in order to achieve its approximate solution. As the exact solution of fractional Kudryashov–Sinelshchikov equation is not recorded earlier in literature, the analytical exact solutions have been constructed by using sech-tanh method via fractional complex transform. The numerical outcomes obtained by RBF are observed to be in good agreement with the exact solutions as well as with the acquired sech-tanh solutions.

1. Introduction

In the recent years, fractional calculus has become the focus of curiosity for many researchers in exclusive disciplines of applied science and engineering because of the fact that a realistic modelling of a physical phenomenon can be efficiently executed by way of utilizing fractional calculus. The investigation of travelling wave solutions for nonlinear fractional order partial differential equations play an important role in the study of nonlinear physical phenomena. It is significant to find new solutions, since either new exact solutions or numerical approximate solutions may provide more information for understanding the physical phenomena. However, a reliable and effective general technique for solving them cannot be found even in the most useful works on fractional derivatives and integrals.

This paper is devoted to find new analytical and numerical solutions of fractional order Kudryashov–Sinelshchikov equation. Because the exact solution of fractional Kudryashov–Sinelshchikov equation is not available in literature, the sech-tanh method has been implemented in order to obtain new solitary wave solutions. But it is not always convenient to use analytical methods for getting exact solutions. So in this paper substantial importance has been given to obtain exact as well as numerical solutions of newly established fractional Kudryashov–Sinelshchikov equation.

* Corresponding author.
E-mail address: santanusaharay@yahoo.com (S. Saha Ray).

http://dx.doi.org/10.1016/j.amc.2016.11.003

Finite difference and finite element methods are well-known techniques for solving partial differential equations. Even though these methods are very effective for solving various kinds of partial differential equations, they provide the solution of the problem on mesh points only and accuracy of the techniques is reduced in nonsmooth and nonregular domains. Also, these methods are quite time-consuming and difficult to use. To avoid the mesh generation, meshless techniques have attracted the attention of researchers these days. In recent years, the radial basis functions method has been used as a powerful meshless procedure for numerical solution of partial differential equations, which is based on the collocation scheme.

In recent years, a newly developed meshless method viz. radial basis functions (RBFs) has drawn the attention of many researchers in science and engineering for the approximate solution of PDEs. In 1990, Kansa [1,2] developed a meshless method, the so-called Kansa's method, which is obtained directly by collocating the RBFs, particularly the multiquadric (MQ), for computing the numerical approximate solution. This Kansa's method was recently extended to solve various ordinary and partial differential equations.

The multiquadric (MQ) was first developed by Hardy [3] in 1971 as a multidimensional scattered interpolation method in modelling of the earth's gravitational field. The traditional RBFs are globally defined functions that result in a full resultant coefficient matrix. In most of the cases, the accuracy of the RBFs solution depends on the choice of a shape parameter c in the MQ or Gaussian basis functions. Many authors have investigated the shape parameter and the optimal choice of shape parameter c is still under intensive investigation. In general, as c increases, the system of equations to be solved becomes ill-conditioned. In order to deal with the ill-conditioning problem, Wendland [4] constructed a new class of compactly supported RBFs.

Our aim in the present work is to develop a new meshless numerical scheme to solve the fractional Kudryashov–Sinelshchikov equation utilising the collocation technique and approximating directly the solution using the multiquadric radial basis function.

Consider the following Kudryashov–Sinelshchikov equation [5] for describing the pressure waves in a mixture liquid and gas bubbles taking into consideration the viscosity of liquid and the heat transfer

$$u_t + \gamma u u_x + u_{xxx} - \varepsilon (u u_{xx})_x - \beta u_x u_{xx} - \nu u_{xx} - \mu (u u_x)_x = 0 \tag{1.1}$$

where u denotes the density and γ, ε, β, ν and μ are real parameters. When $\varepsilon = \beta = \nu = \mu = 0$, Eq. (1.1) reduces to KdV equation. However Eq. (1.1) transformed to Burger-KdV (BKdV) equation when the parameters ε, β and μ are zero. Hence Eq. (1.1) is a generalization of the KdV and the BKdV equation and also similar to the Camassa–Holm (CH) equation. The Kudryashov–Sinelshchikov equation was first proposed by Kudryashov and Sinelshchikov [5] in the year 2010 for describing the pressure waves in a mixture liquid with gas bubbles taking viscosity of liquid and heat transfer into consideration.

Next, we consider the time-fractional Kudryashov–Sinelshchikov equation as

$$D_t^{\alpha} u + \gamma u u_x + u_{xxx} - (u u_{xx})_x - \beta u_x u_{xx} - \nu u_{xx} - \mu (u u_x)_x = 0 \tag{1.2}$$

which is a generalization of Eq. (1.1). Here α denotes the order of Caputo fractional derivative whose range is $0 < \alpha \leq 1$.

The classical Kudryashov–Sinelshchikov has received substantial attention by many authors in recent years. Some new types of solitary and periodic wave solutions for Kudryashov–Sinelshchikov equations are presented in Ref. [6]. Many analytical and numerical methods have been implemented for the study of Kudryashov–Sinelshchikov equation. Various methods such as the bifurcation method of dynamical systems and the method of phase portraits analysis [7,8], multiple G'/G-expansion method [9], Improved F-expansion method [10], Lie symmetry method [11], first integral method [12], and modification of truncated expansion method [13] had been used to acquire exact solutions of Kudryashov–Sinelshchikov equation.

But according to the best possible information of the authors, the detailed study of the time fractional Kudryashov–Sinelshchikov equation is only the beginning. To the best possible information of the authors no numerical prior work has been reported to solve the fractional Kudryashov–Sinelshchikov equation. The present paper emphasises on the application of radial basis function method for solving the time-fractional Kudryashov–Sinelshchikov equation with a view to exhibit the capabilities of this method in handling nonlinear equation. The approximate solutions thus obtained by RBF method are compared with analytical solution obtained by using the sech-tanh method for time-fractional Kudryashov–Sinelshchikov equation.

The structure of the paper is as follows: a formal outline to fractional calculus has been supplied in Section 2 for the particular intend of the paper. In Section 3, the mathematical algorithm of sech-tanh method is proposed. The fundamental concept to radial basis function method is discussed Section 4. In Section 5, the sech-tanh method and RBF approach has been implemented to determine the analytical and numerical solutions for fractional Kudryashov–Sinelshchikov equation. The numerical results and discussions are examined in Section 6 and Section 7 accomplishes the paper.

2. Fractional derivative and integration

Various approaches were utilised to describe the derivatives of fractional order. The fractional calculus involves different definitions of the fractional operators such as Riemann–Liouville fractional derivative, Grünwald–Letnikov fractional derivative, Caputo derivative and Riesz derivative.