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A class of initials-dependent dynamical systems

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ABSTRACT

Nonlinear term is critical for emergence of chaos in autonomous dynamical systems. The sampled time series in chaotic system are dependent on the initial selection of variables, while the attractors are invariant for fixed parameters. In this paper, the dynamical behavior of a class of dynamical system is investigated at fixed parameter region. It is found that the state selection is dependent on the initials and the potential mechanism is discussed. It is confirmed that the system can be switched between stable state, periodical state and even chaotic state by selecting appropriate initials even the parameters are fixed. We think that nonlinear cross terms with higher order could account for the emergence of this behavior. It indicates that initial selection and resetting can be also effective to control some chaotic systems, and these chaotic systems could enhance security for possible secure communication because the chaotic attractor depends on the parameter and initials selection as well. In the case of secure communication, the reconstruction of phase space becomes more difficult because the attractors are changed arbitrarily, thus the safety for secure keys is enhanced. For chaos control, when the initials are reset, the controller can be removed and the system can develop to step into the desired target by itself.

1. Introduction

Chaos is observed in chemical, physical and biological systems, and nonlinear analysis is helpful to understand the dynamical behavior and properties of sampled time series for observable variables from chaotic systems [1–12]. The topics about chaos, hyperchaos and spatiotemporal chaos have been investigated extensively [13,14]. The electrical activities in neuron also show chaotic properties and can be verified in neuron models by setting appropriate parameters and external forcing currents [15–18]. Some researchers prefer to find and design different chaotic, hyperchaotic circuits, dynamical models [19–21]. For example, Azzaz et al. [20] proposed an auto-switched chaotic system and its FPGA implementation was verified. Trejo-Guerra et al. [21] presented a review on the electronic design of chaotic oscillators, the integrated realizations were listed, and the key points for future research on the design of multi-scroll chaotic oscillators were discussed. It is believed that the brain normally works in a chaotic mode, while during attention it shows ordered behavior, as a result, Arama et al. [22] presented a novel model for human memory based on the chaotic dynamics of artificial neural networks. Complex dynamical behaviors are observed in nonlinear dynamical systems for economic models, Tacha et al. [23] presented a scheme adaptive control to regulate the finance system's behavior. Fractional chaotic systems seem to present more complex dynamical behavior, Xu et al. [24] dealed with a synchronization scheme for two fractional chaotic

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systems and its possible application on image encryption was discussed. Mata-Machuca [25] dealed with the synchronization and parameter estimations of an uncertain Rikitake system and its application in secure communications employing chaotic parameter modulation was also discussed. The gains of the receiver system were adjusted continually according to a convenient high order sliding-mode adaptive controller (HOSMAC), until the measurable output errors converged to zero. Ghosh and Chowdhury [26] introduced an adaptive learning rule for estimating all unknown parameters of delay dynamical system using a scalar time series, and Krasovskii–Lyapunov theory was used to derive sufficient condition for synchronization.

More researchers would like to explore more effective schemes to suppress chaos, parameter estimation and realize synchronization between chaotic systems [27–36]. Indeed, many chaotic dynamical systems have been set up for bifurcation analysis and synchronization control. Some researchers thought that fractional-order chaotic systems could be much interesting and more important to be worthy of investigation, for example, Zhou et al. [37,38] investigated the stabilization and synchronization on fractional-order chaotic systems with fractional-order $1 < q < 2$ by using adaptive scheme. It is known that chaotic systems are much dependent on the initials selection and the time series or orbits show much diversity even slight difference occurs in the initial values. However, the attractor and attracted basin could be invariable when the bifurcation parameters for the chaotic system are fixed. These known chaotic and hyperchaotic systems can generate finite attractors while some chaotic systems can produce infinite attractors under appropriate control scheme, for example, jerk circuit [39,40] can be controlled to generate a large number of attractors by applying periodical signal forcing [41]. The dynamical properties can also be discerned by estimating the Hamilton energy [42] on these dimensionless dynamical systems, and it is found that chaotic systems with multi-attractors [43] can hold smaller Hamilton energy and neurons under bursting state also hold smaller Hamilton energy as well. It has been confirmed that nonlinear term is important and necessary for nonlinear dynamical system so that chaotic state can be triggered under appropriate parameters selection. In fact, coupled oscillators and networks can present more complex spatiotemporal dynamics and it is important to explore effective schemes that spatiotemporal chaos can be suppressed and synchronization can be realized in complex network [44–46]. In practical verification, many realistic factors should be considered, for example, the effect of time delay, the control cost (power consumption of controller), as a result, intermittent schemes are used to reach this target. Under the framework of Filippov systems and a linear controller, the exponential synchronization and anti-synchronization criteria for memristor-based neural networks can be guaranteed by the matrix measure and Halanay inequality [47]. Mathiyalagan et al. [48] investigated the impulsive synchronization of memristor based bidirectional associative memory (BAM) neural networks with time varying delays. Then the impulsive time dependent results are derived for the exponential stability of the error system by using linear matrix inequality (LMI) approach.

Nonlinear circuits [49–53] are useful to investigate the chaotic problems, and many researchers thought chaotic circuits could be useful for secure communication and image encryption [54–57]. In fact, nonlinear devices such as negative resistance, negative conductor, negative capacitor are important devices for a setting a chaotic circuit. It is important to mention another important device, memristor [58,59], which the memductance is dependent on the external forcing current and thus it is initial-dependent. As a result, the memristor-coupled oscillators hold more complex dynamical behaviors. The circuit composed of memristor is dependent on the bifurcation parameter and also the initials selection [60]. For most of the well-known chaotic systems, the attractors and basin of attracts keep invariable when the parameters are fixed though the output time series can show some differences by setting different initial values for variables. Therefore, it is interesting to investigate these dynamical systems and its potential mechanism why the developed state also depends on the initial selection. In fact, the initial-independence is associated with memory, it is confirmed that memristor-coupled circuit or oscillator can be switched between different kinds of attractors by resetting the initials. Can we find an effective scheme to develop more chaotic systems which are dependent on initials selection and parameter setting? In this paper, we argue that initial dependence could be associated with nonlinear cubic terms composed of different variables, the potential mechanism on other chaotic systems are discussed. Firstly, the memristor-coupled oscillator is used for preliminary discussion. Secondly, the Rössler model is improved by adding quadratic term into the dynamical system, and the spectrum of Lyapunov exponents, phase portrait and basin of attractor are calculated to discern the dependence of initials selection on attractors.

2. Model, scheme and discussion

It is known that the memductance for memristor is dependent on the initial inputs, as a result, the nonlinear circuits or systems composed of memristor could be dependent on the initial selection for magnetic flux across the memristor. At first, we investigate the dynamics for a class of circuit composed memristor and then explore the potential mechanism for this initial-dependent property on other nonlinear systems so that desired nonlinear circuits can be set up for possible application for secure communication and control. The memristor-coupled circuit [61–63] can be illustrated in Fig. 1. The outputs of circuit can be described by nonlinear equations according to Kirchhoff's law as follows