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Stability and Stabilization of Nonlinear Switched Systems Under Average Dwell Time

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ABSTRACT

In this paper, the problems of stability and stabilization of nonlinear discrete-time switched systems is investigated. Firstly, in order to implement the lower bound of minimum average dwell time (ADT) of discrete-time switched nonlinear systems, the ι -open-chain and quasi-cyclic switching signals are introduced. Secondly, the problem of these underlying nonlinear discrete-time switched systems are solved by using the interval type-2 (IT2) fuzzy modeling approach. Thirdly, a novel delayed IT2 fuzzy controller is devised to guarantee the asymptotically stable of the resulting systems. Finally, a numerical simulation example is given to show the merit and effectiveness of the proposed approach.

1. Introduction

Switched systems [1] have been widely applied in various fields such as power electronics, power systems, chemical processes and network communications. One of the most significant problems for switched systems is stability. It is mainly divided into two categories: the stability under arbitrary switching [2] and under constrained switching [3–7]. ADT switching is a kind of typical constrained switching. The problem of filter design for a class of switched system with ADT switching has been concerned in [8]. By using the piecewise Lyapunov function technique, the authors in [9] have investigated the problem of exponential stability analysis of switched delayed neural networks under ADT switching. [10] addressed the issues of feasibility, stability and robustness on the switched model predictive control. In [11], the authors proposed an elementary time unit approach to study the problem of estimating the admissible delay in switched systems.

On the other hand, in handling the problem of modeling and control for the stability of nonlinear systems, approximation-based fuzzy or neural adaptive back stepping control [12–24] plays an important role. Takagi–Sugeno (T–S) fuzzy model [25–31] is used to approximate any smooth nonlinear systems with arbitrary accuracy on set. Recently, based on the T–S fuzzy control approach, nonlinear switched systems have been investigated [32–37]. In order to derive the stabilization criteria of the switched fuzzy system, the authors transform T–S fuzzy system into an equivalent switched fuzzy system corresponding to each subregion by piecewise Lyapunov function method in [38]. In [39], the problems of exponential stability and asynchronous stabilization for switched nonlinear systems have been studied by T–S fuzzy model approach. Moreover, it should be pointed out that the method of handling uncertainty of nonlinear systems is not effective. IT2 fuzzy logic systems have been designed and implemented in [40,41]. Then, to facilitate stability analysis of the IT2 fuzzy model, the

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IT2 fuzzy controller of the footprint of uncertainty has been used to develop some membership function conditions, which allow the introduction of slack matrices to handle the parameter uncertainties in the stability analysis in [42]. The author of [43] proposed a novel IT2 fuzzy controller, where the membership functions and number of rules can be freely chosen and different from those of the IT2 T–S fuzzy model to deal with nonlinear systems subject to parameter uncertainties. Broadly speaking, [42,43] focus on investigating the universal nonlinear systems. But this paper studies the discrete-time switched nonlinear IT2 fuzzy systems.

In this paper, the problems of stability and stabilization for discrete-time switched IT2 fuzzy systems will be studied. The key contributions of the paper consist of: 1) Based on the novel concepts of ι-open-chain and quasi-cyclic switching signal, the lower bound of minimum ADT is obtained. 2) A novel switched IT2 fuzzy system is established, which allows the Lyapunov-like function to increase in some interval of active subsystems and decrease in the other interval of active subsystems. It will remain stabilization by taking delay controlling to close the feedback loop. 3) The stability and stabilization conditions can reduce the conservatism of the switched IT2 fuzzy system by introducing more slack matrices.

Notations: The notations are fairly standard through the paper. $\mathbb{R}^n$ and $\mathbb{R}^{n\times n}$ denote the field of real numbers, n-dimensional Euclidean space and the space of $n \times n$ matrices with real entries, respectively. The notation $\|\cdot\|$ refers to the Euclidean norm. A matrix whose dimension is not explicitly stated is assumed to be compatible for algebraic operations. For a number set X with finite elements $max(x)$ ($min(x)$) means the maximum (minimum) value of X. A function $\alpha : [0, \infty) \rightarrow [0, \infty)$ is said to be of class $\mathcal{K}$ if it is continuous, strictly increasing with $\alpha(0) = 0$. Class $\mathcal{K}_\infty$ denotes the subset of $\mathcal{K}$ consisting of all those unbounded functions. A function $\beta : [0, \infty) \times [0, \infty) \rightarrow [0, \infty)$ is said to be of class $\mathcal{KL}$ if $\beta(\cdot, t)$ is of class $\mathcal{K}$ for each fixed $t > 0$ and $\beta(r, t)$ is decreasing to zero as $t \rightarrow \infty$ for each fixed $r \geq 0$. A^T stands for the transpose of matrix A, and $P > 0$ (≥ 0) denotes that P is a real symmetric and positive definite (semi-positive definite) matrix.

2. Problem formulation

Consider the following discrete-time switched system:

$$x(k+1) = f_{\sigma(k)}(x(k), k), \quad x(k_0) = x_0, \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the system state and $x(k_0)$ is initial state, k_0 denotes the initial time step, $\sigma(k)$ is a switched signal in $p = \{1, 2, \cdots m\} \in \mathcal{I_N}$, where the number of switching subsystems is activated. $f_{\sigma(k)} : \mathbb{R}^n \times \mathbb{R} \rightarrow \mathbb{R}^n$ are smooth functions for any $\sigma(k) = p$, and it may be autonomous and controlled in the switched sequence. The switched sequence is $0 < k(1) < k(2) < \cdots < k(p) < k < k(p+1) < \cdots$.

2.1. Switched IT2 T–S fuzzy model

The discrete-time switching IT2 T–S fuzzy system is employed to depict system (1) as follows:

If $f_1(x(k))$ is $\widetilde{N}^i_{p1}$, and, …, and $f_\psi(x(k))$ is $\widetilde{N}^i_{p\psi}$, Then

$x(k+1) = A_{pi}x(k) + B_{pi}u(k)$,

where $\widetilde{N}^i_{p\alpha}$ denotes the p^{th} switched subsystem corresponding to the i^{th} IT2 fuzzy set function $f_\alpha(x(k))$, $\alpha = 1, 2, \ldots, \psi$ and $i \in \varphi \triangleq \{1, 2, \ldots, r\}$; ψ is a positive integer; $A_{pi} \in \mathbb{R}^{n\times n}$ and $B_{pi} \in \mathbb{R}^{n\times m}$ are known constant system matrices; $x(k) \in \mathbb{R}^n$ stands for the system state vector; $u(k) \in \mathbb{R}^m$ denotes the system input vector. The i^{th} strength of the activation is the following interval sets: $\widetilde{w}_{pi}(x(t)) = [\underline{w}_{pi}(x(k)), \overline{w}_{pi}(x(k))]$, $i = 1, 2, \cdots, r$,

$$\underline{w}_{pi} = \underline{\mu}_{\widetilde{M}^i_{p1}}(f_1(x(k))) \times \underline{\mu}_{\widetilde{M}^i_{p2}}(f_2(x(k))) \times \cdots \times \underline{\mu}_{\widetilde{M}^i_{p\psi}}(f_\psi(x(k))) \geq 0,$$
$$\overline{w}_{pi} = \overline{\mu}_{\widetilde{M}^i_{p1}}(f_1(x(k))) \times \overline{\mu}_{\widetilde{M}^i_{p2}}(f_2(x(k))) \times \cdots \times \overline{\mu}_{\widetilde{M}^i_{p\psi}}(f_\psi(x(k))) \geq 0,$$

where the lower grade of membership (LGM) $\underline{\mu}_{\widetilde{M}^i_{p\alpha}}(f_\alpha(x(k))) \in [0, 1]$ is dictated by the lower membership functions (LMFs) and upper grade of membership (UGM) is $\overline{\mu}_{\widetilde{M}^i_{p\alpha}}(f_\alpha(x(k))) \in [0, 1]$ is governed by the upper membership functions (UMFs). The property $\overline{\mu}_{\widetilde{M}^i_{p\alpha}}(f_\alpha(x(k))) \geq \underline{\mu}_{\widetilde{M}^i_{p\alpha}}(f_\alpha(x(k)))$ leading to $\overline{w}_{pi}(x(k)) \geq \underline{w}_{pi}(x(k))$ for all i. The discrete switched IT2 T–S fuzzy system is described by:

$$x(k+1) = \sum_{p=1}^{m}\sum_{i=1}^{r} w_{pi}(x(k))\big(A_{pi}x(k) + B_{pi}u(k)\big),$$
$$i \in \varphi = \{1, 2, \cdots, r\}, \; p \in S, \tag{2}$$

where

$$w_{pi}(x(k)) = \frac{\widetilde{w}_{pi}(x(k))}{\sum_{i=1}^{r} \widetilde{w}_{pi}(x(k))},$$
$$\widetilde{w}_{pi}(x(k)) = \underline{w}_{pi}\underline{v}_{pi} + \overline{w}_{pi}\overline{v}_{pi}(x_1),$$