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Computational methods for solving the steady flow of a third grade fluid in a porous half space

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ABSTRACT

An important class of fluids commonly used in industries is non-Newtonian fluids. In this paper, two numerical techniques based on rational Legendre functions and Chebyshev polynomials are presented for solving the flow of a third-grade fluid in a porous half space. This problem can be reduced to a nonlinear two-point boundary value problem on semi-infinite interval. Our methods are utilized to reduce the computation of this problem to some algebraic equations. The comparison of the results with the other methods and residual norm show very good accuracy and rate of convergence of our approach.

1. Introduction

During the past few decades, a lot of attention has been devoted to the study of non-Newtonian fluids because of their practical importance in engineering and industry. Such fluids occur in blood, tomato ketchup, honey, certain oils and greases, mud, paints, plastics and polymer solutions [1,2]. The governing equations for these fluids are more non-linear and higher order than those of Newtonian fluids [3–6]. In addition, analytic solutions of the most equations involving non-Newtonian fluids cannot be obtained explicitly [3,7,8]. So, numerical solution of these equations is of practical importance. The steady state flow of a third grade fluid in a porous half space can be modeled by the following nonlinear boundary value problem [9]

$$\frac{d^2 f}{dz^2} + b_1 \left(\frac{df}{dz}\right)^2 \frac{d^2 f}{dz^2} - b_2 f \left(\frac{df}{dz}\right)^2 - cf = 0, \tag{1}$$

$$f(0) = 1, \qquad f(\infty) = 0. \tag{2}$$

Where

$$b_1 = \frac{6\beta V_0^4}{\mu \nu^2}, \quad b_2 = \frac{2\beta \varphi V_0^2}{k\mu}, \quad c = \frac{\varphi \nu^2}{k V_0^2}, \quad f(z) = \frac{u}{V_0}, \quad z = \frac{V_0}{\nu} y.$$

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Here, μ is the dynamic viscosity, ρ is the fluid density, $\nu = \mu/\rho$ is the kinematic viscosity, u represents the fluid velocity, k and φ, respectively represent the permeability and porosity of the porous half space which occupies the region $y > 0$. Also $V_0 = u(0)$ and β is material constants. Note that, $b_2 = b_1 c/3$.

Different techniques have been used to obtain analytical and numerical solutions for this problem. In [9] homotopy analysis method is applied to solve second-order nonlinear boundary value problem (1)–(2). Ahmad in [10] gave the asymptotic form of the solution and utilized this information to develop a series solution. Also, a collocation method based on radial basis functions is presented by Kazem et al. [11]. Moreover, authors of [12] solved this problem using the collocation method based on modified generalized Laguerre functions. Recently, authors in [13] solved this problem by Tau method. In Tau method they applied the operational matrices of derivative and product of rational and exponential Legendre functions together to reduce the solution of this problem to the solution of a system of algebraic equations.

The main purpose of the present paper is to develop rational Legendre collocation(RLC) method and Chebyshev finite difference (ChFD) method for numerical solution of this problem. Our approach consists of reducing the problem to a set of algebraic equations by expanding $f(z)$ in terms of either rational Legendre functions or Chebyshev polynomials with unknown coefficients. It is known that the rational Legendre functions are a complete spectral basis for the semi-infinite interval [14–16]. These functions play an important role in recent research works for solving various kinds of problems in semi-infinite interval. Authors of [15] applied a spectral scheme using these orthogonal functions for solving the Korteweg-de Vries equation on the half line. Also, Parand et al. [17–19] and Tajvidi et al. [20] applied rational Legendre and Chebyshev functions with tau and collocation method to solve nonlinear ordinary differential equations on semi-infinite intervals. For more research works on the rational Legendre functions, we refer the interested reader to [16], and the references therein.

Moreover, ChFD method is a nice and powerful approach for numerical solution of linear and nonlinear differential equations. In ChFD method we used Chebyshev expansions to approximate $f(z)$ and employed the Chebyshev–Gauss–Lobatto points as the interpolating points [21,22]. The Chebyshev–Gauss–Lobatto points have explicit forms and can be obtained easily. This method can be regarded as a non-uniform finite difference scheme and is more accurate in comparison to the finite difference method [21]. Over the last two decades, ChFD method has been successfully used to solve a wide variety of problems. This method has been extended to handle a system of second-order boundary value problems [23], problems in calculus of variation [24], a problem arising from chemical reactor theory [25], Fredholm integro-differential equation [26], the problem of heat transfer to MHD flow of a micropolar fluid [27], boundary value problems [21] and heat transfer problem [22].

The rest of the current paper is categorized as follows: In the next section, we give details of implementation of RLC method. In Section 3, we describe ChFD method. In Section 4, some numerical results are given to clarify the RLC and ChFD methods. Also, we compare our achievements with existing results in the literature and show how accurate our results are. Section 5 is devoted to conclusions.

2. Rational Legendre collocation method

2.1. Rational Legendre functions

The rational Legendre function of degree n, with the map parameter $L > 0$, is defined by [14,16]

$$R_n(x) = P_n\left(\frac{x-L}{x+L}\right), \tag{3}$$

in which, $P_n(x)$ is nth Legendre polynomial. In fact, this transformation is chosen such that it maps the finite interval $[-1, 1]$ onto $I = [0, \infty)$. Boyd [14,28] presented some algebraic maps for every fixed L. Rational Legendre functions satisfy the recurrence relation:

$$R_0(x) = 1, \quad R_1(x) = \frac{x-L}{x+L},$$
$$R_{n+1}(x) = \left(\frac{2n+1}{n+1}\right)\frac{x-L}{x+L}R_n(x) - \left(\frac{n}{n+1}\right)R_{n-1}(x), \quad n \geqslant 1.$$

Also, $R_n(x)$ is the nth eigenfunction of the singular Sturm–Liouville problem

$$\frac{(x+L)^2}{L}\partial_x(x\partial_x R_n(x)) + n(n+1)R_n(x) = 0, \quad x \in [0, \infty), \quad n = 0, 1, 2, \cdots \tag{5}$$

Now, let us define

$$L^2_\omega(I) = \{f : I \to \mathbb{R} \mid f \text{ is measurable and } \|f\|_\omega < \infty\}, \tag{6}$$

where $\omega(x) = 2L/(x+L)^2$ is a non-negative, integrable, real-valued function over the interval I and