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Analysis of weak solution of Euler–Bernoulli beam with axial force

Bidisha Kundu, Ranjan Ganguli*

Department of Aerospace Engineering, Indian Institute of Science, Bangalore 560012, India

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ABSTRACT

In this paper, we discuss about the existence and uniqueness of the weak form of the non-uniform cantilever Euler–Bernoulli beam equation with variable axial (tensile and compressive) force. We investigate the reason of the buckling from the coercivity analysis. The frequencies of the beam with tensile force are found by the Galerkin method in the Sobolev space H^2 with proper norm. Using this method, a system of ordinary differential equations in time variable is formed and the corresponding mass and stiffness matrices are constructed. A very general form of these matrices, which is very simple and suitable for calculations, is derived here with a standard basis. Numerical results for rotating beams with polynomial stiffness and mass variation, typical of wind turbine and helicopter rotor blades, are obtained. These results match well with the published literature. A new polynomial generating set is found. Using two elements of this set, a formula to find the eigenfrequencies is derived. The proposed approach is easy to implement in symbolic computing software.

1. Introduction

For the last five decades, the rotating cantilever beam model has drawn the attention of researchers because of its widespread applications in the engineering field such as for helicopter rotor blades, wind turbine blades, propeller blades, gas turbine blades, flexible robot arms etc. The Euler–Bernoulli beam theory is typically used to model such rotating structures. We should calculate the exact fundamental frequencies of these structures to design them to avoid the resonance and vibration problem [1]. Unfortunately, the exact solution of the response of the rotating beam is very onerous [2]. Therefore, the search for an approximate solution abounds in the literature. Among these approximate methods, the Rayleigh–Ritz method [3–5], the Galerkin method [6,7] and the finite element method [8–12] are very famous.

Due to the paucity of exact solution, the solution generated from one approximate method is compared with another standard approximate method or a semi-analytical method such as the Frobenious method [13]. The Frobenious method gives very good results as it is based on the series solution of the governing differential equation. The dynamic stiffness method is also very effective for solving the free vibration problem. Using these methods, we can control the level of accuracy to any desired extent [14–17].

There is another very interesting method—Inverse problem method. If frequency and mode shape of a beam are given, then this method helps us to find the stiffness and mass of this system. The first successful application of this method was

* Corresponding author. Fax: 91 80 2360 0134.
E-mail addresses: bidisha@aero.iisc.ernet.in (B. Kundu), ganguli@aero.iisc.ernet.in, drganguli@gmail.com (R. Ganguli).

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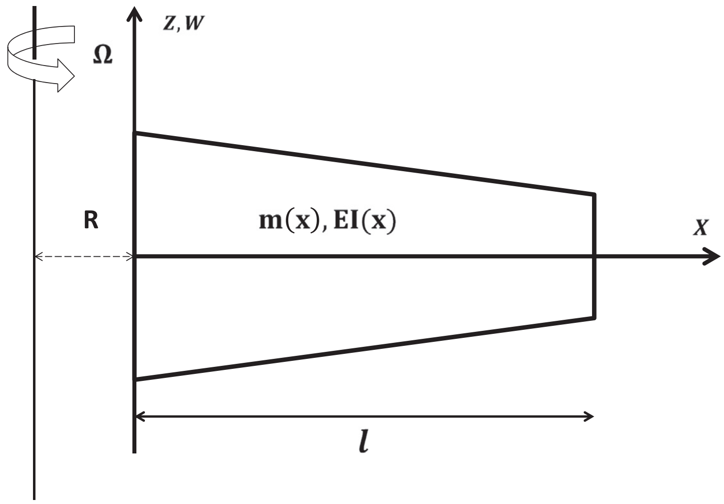

Fig. 1. Schematic diagram of a rotating beam.

made by Elishakoff and Candan [18] on the inhomogeneous beam with different boundary conditions. Sarkar and Ganguli [19] found a collection of nonuniform Euler–Bernoulli rotating beams of different stiffness and mass variation having same fundamental frequencies by using the inverse problem method.

As the direct exact solution of the non-homogeneous beam with rotation is unavailable except for some peculiar special cases [20], indirect methods like isospectral method are created. In this method, two different types of beams having the same eigenfrequencies, are sought. Kambampati et al. [21] found the nonuniform rotating beam isospectral to the uniform non-rotating beam. Since uniform non-rotating beam have an exact solution, the isospectral rotating beams could be used as benchmark solution to test finite element codes.

The finite element method is used extensively in different forms to solve the equation of the rotating beam. The modified finite element analysis, like h-version, p-version, Fourier-p, dynamic finite element and spectral element method [22–24], are created to make the solution procedure more efficient. In some papers [25,26], the Galerkin method is applied on a finite dimensional subspace of the solution space to get the approximate solution. The Galerkin method and the finite element method are very congruent since they originated from the weak formulation of the physical problem. So we should investigate the analytical area of this numerical technique for non-uniform rotating beam model. The main motivation of this investigation is to establish a strong analytical foundation of this numerical technique to control the error and the accuracy.

Here, we discuss about the mathematical analysis of non-uniform Euler–Bernoulli beam with axial force, define its weak form and check the existence and uniqueness of the solution using the Galerkin method. This method has been applied on several physical problems [27] efficiently and its mathematical properties have been studied. To prove the existence and uniqueness of our weak problem, we make some assumptions which are related to the stiffness and rotating force of the system. For the beam with compressive axial force case, the net stiffness of the beam is decreasing which may introduce some physical problem such as buckling in the system. To introduce this difficulty in mathematical form, we establish a relation between stiffness and compressive force to get the proper approximate solution. In [28–30], the existence of general elastic problem is discussed which helps us to define the weak form. After defining the weak form, we test the existence using the Galerkin method and then we search our solution in finite dimensional subspace to find the approximate solution. The approximate solution depends on the stiffness and axial force via the coefficient matrix generated from the Galerkin process. We investigate how the nature of the solution depends on the physical properties of the beam.

The beam with axial force has many practical applications in engineering field. In the paper [31], these types of systems are studied. The research on the beam with rotation (tensile axial force) is still developing [32] and different types of problems related with beam with rotation are being studied [33,34] for application in the engineering field. Hence we study both types of beams here i.e., the beam with axial tension and axial compression. We check the existence of weak solution simultaneously for these two types of beams.

2. The rotating Euler–Bernoulli beam

In this section we introduce the dynamic evolution equation of motion of the rotating Euler–Bernoulli beam (REBB) equation, a very important example of the beam with tensile axial force, shown in Fig. 1. The beam is of varying stiffness and varying mass where $EI(x)$ and $m(x)$ are the flexure stiffness and mass per unit length at a distance “x” from the axis of rotation. Here we fix the coordinate axes X, Y, Z along the length, breath and height of the beam, respectively. We consider the beam is slender and of length l and it is rotating with constant angular speed “Ω” with respect to Z. We denote by $W(x, t)$, $M(x, t)$ and $f(x, t)$, the out-plane bending displacement along Z, bending moment and the external force, respectively, at