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Magneto-hydrodynamics-driven mixing of a reagent and a phosphate-buffered solution: A computational study

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ABSTRACT

Magnetohydrodynamic (MHD) mixing, which is one of the most active mixing methods in a microfluidic system, can be used to optimize the mixing of a reagent and phosphate-buffered solution (PBS) within a short time. The aim of this study is to investigate the capability of MHD mixing with respect to the shape and configuration of the electrodes, the applied voltage, and the height of the micromixer. A reagent that fills the mixer is considered for mixing with the PBS. The mixing capabilities of six different electrode configurations are first quantitatively evaluated based on a mixing index. The configuration determined to be the most effective is then used to evaluate the mixing capability with respect to the applied voltage and height of the micromixer. The results of this study confirm that numerical analysis can be used to determine the optimal MHD mixing conditions for various electrode geometries.

1. Introduction

Recent developments of microelectromechanical systems (MEMSs) have promoted the application of microfluidic systems such as the micro-total-analysis-systems (μ-TAS) in the diagnostic field [1–3]. A microfluidic system mainly consists of a micropump and a microvalve for fluid transport, a micromixer for blending fluids, and a microsensor for fluid analysis and detection [1,2,4]. It is particularly necessary for the micromixer, which is used to blend a reagent and a sample to promote biochemical reaction in preparation for analysis and diagnosis, to perform the mixing as efficiently as possible, within a short time [2,5,6]. However, fluid micromixing by diffusion requires a relatively long time because of the low diffusivity of most fluids, and it is impossible to enhance the mixing by turbulent flow owing to the characteristic micrometer-scale length.

The mixing methods employed in micromixers can be classified based on energy application as passive mixing (without external energy) and active mixing (with external energy) [5]. In most passive methods, the channel is designed to increase the contact surface and reduce the diffusion length, where the fluids are mixed while they flow through the channel. Passive methods can be further classified based on the system geometry such as lamination [4], intersecting channels [7], serpentine channel [8,9], embedded barriers [10], slanted wells [11], and twisted channels [12]. Active methods, which enable faster fluid mixing, can be classified based on the applied energy such as pressure-field-driven mixing [13], acoustic (supersonic)-driven mixing [14], temperature-induced mixing [15], dielectrophoretic mixing [16], electrokinetic instability mixing [17],

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electrohydrodynamic mixing [18], and magnetohydrodynamic (MHD) mixing [19–27]. Among these active mixing methods, we consider the MHD mixing in the present study. In MHD mixing, the fluid flow is controlled by inducing a Lorentz force in an electrically conducting fluid [28,29]. The advantages of MHD micromixing include the requirement of a small voltage to control the fluid flow, and the ability to change the direction of the Lorentz force by changing the polarity of the electrodes. However, if the voltage difference between the electrodes is too large, bubbles would be formed on the electrodes by electrolysis or unwanted chemical reaction, and this would disturb the mixing which may also result in the generation of excess heat [23].

Numerous studies have been conducted on the inducement of chaotic flow in MHD micromixers (see Table 1). Bau et al. [19] proposed a hexahedral MHD micromixer with the electrodes located at the bottom wall; they investigated the flow in the mixer both theoretically and experimentally. Yi et al. [25] studied the flow in a cylindrical MHD micromixer with two electrodes placed at different positions with periodic variation of the voltage. Qian et al. [22] theoretically analyzed the flow in an MHD micromixer with electrode sets that were periodically turned on and off; they observed the mixing behavior by flow visualization. There have also been recent reports on the use of computational fluid dynamics (CFD) approach to evaluate the mixing capabilities of MHD micromixers. For example, Yuan and Isaac [26] numerically investigated the mixing capability of 2-D circular MHD micromixers with various electrode configurations operated by an alternating current. La et al. [21] also numerically analyzed the steady-state flows through T-shaped microchannels that were used to pump and mix fluids by the application of voltages to electrodes on the side and bottom walls, and evaluated the mixing capability using the standard deviation of the PBS Fraction at the outlets. Qian and Bau [24] numerically analyzed an MHD micromixer and experimentally evaluated the flow. Kang and Choi [20] used numerical simulations and also performed experiments to investigate the changes in the mixing capability of an MHD micropump with respect to the shape of the electrodes and the applied voltage.

The present study focuses on the enhancement of the mixing capability of an MHD micromixer without an inlet or outlet. A CFD approach is used to evaluate the effects of the electrode configuration, applied voltage, and height of the micromixer on the mixing performance, with the purpose of maximizing the mixing of a reagent with an initial thickness of 25 μm. In Sections 2 and 3, we present the governing equations and numerical methods that are used to predict the mixing performance, as well as the considered electrode configurations. In Sections 4 and 5, we discuss the results.

2. Governing equations and constitutive relations

To predict the mixing capability of an MHD micromixer, in the absence of any thermal effects, the governing equations include the conservation of mass:

$$\frac{\partial \rho}{\partial t} + \mathrm{div}(\rho \mathbf{u}) = 0 \tag{1}$$

And the linear momentum equation including the Lorentz force term (see Davidson [28], Lorrain et al. [29], and Wang [30]):

$$\frac{\mathrm{d}\mathbf{u}}{\mathrm{d}t} = \mathrm{div}\,\mathbf{T} + \rho \mathbf{b} + \mathbf{F}_\mathrm{L} \tag{2}$$

where $\frac{\mathrm{d}(\cdot)}{\mathrm{d}t} = \frac{\partial(\cdot)}{\partial t} + [\mathrm{grad}(\cdot)]\,\mathbf{u}$, ρ is the density of the conducting fluid, $\mathbf{u}$ is the velocity vector, $\mathbf{T}$ the stress tensor, $\mathbf{b}$ the body force, and $\mathbf{F}_\mathrm{L}$ the Lorentz force caused by the electric current and magnetic field interaction. In the above equations, constitutive equations are needed for the stress tensor $\mathbf{T}$ and the Lorentz force $\mathbf{F}_\mathrm{L}$, respectively. We assume the fluid is an incompressible linear viscous conducting fluid. As a result, the conservation of mass reduces to

$$\mathrm{div}(\mathbf{u}) = 0 \tag{3}$$

And the stress tensor is given by

$$\mathbf{T} = -\mathrm{p}\mathbf{I} + 2\mu \mathbf{D} \tag{4}$$

where p is the pressure, μ is the coefficient of shear viscosity, and $\mathbf{D} = \frac{1}{2}[\mathrm{grad}\ \mathbf{u} + (\mathrm{grad}\ \mathbf{u})^\mathrm{T}]$ is the symmetric part of the velocity gradient. Furthermore, we are assuming that the application of the magnetic or the electric field do not change the rheological behavior of the fluid and the influence of the magnetic field on the fluid is only through the presence of the Lorentz force. In general, however, an electromagnetic field can cause changes in the behavior of fluid and in that case one needs to consider the mechanics of electrorheological or magnetorheological fluid. For a discussion of those issues, see Rajagopal and Ruzicka [31], Ruzicka [32], Bullough [33], Wineman and Rajagopal [34], and Dorfmann et al. [35]. The third term on the right-hand side of Eq. (2) is the Lorentz force term, and is defined as (Bau et al. [19]; La et al. [21]; Qian and Bau [24]; Wang [30]):

$$\mathbf{F}_\mathrm{L} = \mathbf{J} \times \mathbf{B} \tag{5}$$

where $\mathbf{J}$ and $\boldsymbol{B}$ are the electric current density and the magnetic induction, respectively. We assume that the magnetic permeability of PBS is sufficiently small so that the magnetic field inside the PBS solution can be approximated with the