Applied Mathematics and Computation 298 (2017) 296–309

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Stability analysis of fractional-order complex-valued neural networks with both leakage and discrete delays

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ARTICLE INFO

Keywords:
Fractional-order complex-valued neural networks
Uniform stability
Leakage delay
Discrete delay

ABSTRACT

In this paper, the problem of uniform stability for fractional-order complex-valued neural networks with both leakage and discrete delays is considered. Base on the contracting mapping principle, a sufficient condition is proposed for the existence and uniqueness of the equilibrium point of the addressed neural networks. By employing analysis technique, some delay-dependent criteria are established for checking the global uniform stability of the fractional-order complex-valued neural networks. Two numerical examples are presented to demonstrate the validity and feasibility of the proposed results.

1. Introduction

As we known, as an extension of real-valued neural networks [1–7], complex-valued neural networks with complex-valued state, output, connection weight, and activation function have been one of the most extensively investigate topic in many research areas. This is mainly due to their widespread applications in physical systems dealing with electromagnetic, light, ultrasonic, quantum waves and so on. For example, see [8–11], and the references therein. These applications depend on the dynamical behaviors of the system heavily. Therefore, more and more attentions have been paid to the study on the dynamic behaviors of complex-valued neural networks [12–17].

It is recognized that fractional-order models have showed more merits than classical integer-order models in describing the hereditary and memory properties for various materials and processes [18,19]. Currently, fractional calculus has already been introduced into neural models by many researchers. In [20], several topics related to the dynamics of fractional-order neural networks of Hopfield type are investigated, such as stability and multi-stability (coexistence of several different stable states), bifurcations and chaos. In [21], the existence and α-exponential stability of the equilibrium point of a class of fractional-order neural networks are considered. Besides, the α-synchronization of fractional chaotic networks is proposed by imposing a linear control. In [22], some new sufficient conditions are established to ensure the existence and uniqueness of the nontrivial solution for a general class of neural networks with a fractional-order derivative. Moreover, uniform stability of the fractional-order neural networks is proposed in fixed time-intervals. In [23], the conditions on the global Mittag–Leffler stability and synchronization are established by using Lyapunov method for the memristor-based fractional-order neural networks. In [24], by combining a fractional-order differential inequality, some sufficient conditions are derived for the projective synchronization of fractional-order memristor-based neural networks. In [25], by applying Lyapunov approach, linear state feedback control law and partial state feedback control law are presented to stabilize the fractional-order bidirectional associative memory neural networks. In [26], under the framework of Filippov solutions, a growth condition is

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http://dx.doi.org/10.1016/j.amc.2016.11.027
0096-3003/

given to guarantee the existence of the solutions for fractional-order Hopfield neural networks with discontinuous activation functions. Besides, some sufficient conditions are proposed for the boundedness and stability of the solutions of such discontinuous networks by employing the Lyapunov functionals. In [27], based on the nonsmooth analysis and control theory, some sufficient criteria for the global Mittag–Leffler synchronization of a class of fractional-order neural networks with discontinuous activations are derived by designing a suitable controller.

It is well known that time delay is commonly encountered in biological and artificial neural networks. It has been found that, the existence of time delays often causes undesirable dynamic behaviors such as performance degradation, oscillation, or even instability of the systems. Therefore, stability analysis of complex-valued neural networks with time delays has received much attention and various stability conditions have been obtained [28–38,51,52]. On the other hand, recently some stability results about delayed fractional neural models have been derived. In [39], the authors investigate the uniform stability for a class of fractional-order neural networks with constant delay by the analytical approach. In [40], a finite-time stability criterion for Caputo fractional neural networks with distributed delay is established by using the theory of fractional calculus and generalized Gronwall–Bellman inequality approach. In [41], by using the contracting mapping principle, method of iteration and inequality techniques, a sufficient condition is established to ensure the existence, uniqueness and finite-time stability of the equilibrium point of the fractional-order neural networks with delay. In [42], the global stability analysis of fractional-order Hopfield neural networks with time delay is investigated. In [43], the global asymptotic stability and synchronization of a class of fractional-order memristor-based delayed neural networks are investigated. In [44], the authors research the global $O(t^{-\alpha})$ stability and the global asymptotical periodicity for a non-autonomous fractional-order neural networks with time-varying delays. It should be noted that the delayed fractional-order neural networks mentioned above are all considered in real domain. However, there are few works on the stability of fractional-order neural networks with time delays in complex domain [45,46]. In [45], the authors deal with the problem of existence and uniform stability analysis of fractional-order complex-valued neural networks with constant time delay. In [46], the problem of the global $O(t^{-\alpha})$ stability and global asymptotic periodicity for a class of fractional-order complex-valued neural networks with time varying delays is investigated.

Motivated by the above discussions, in this paper, we will consider the problem of uniform stability for fractional-order complex-valued neural networks with both leakage and discrete delays. Meanwhile, the existence, uniqueness, and the global uniform stability of equilibrium point of the considered neural networks are researched. Finally, two numerical examples are given to illustrate the effectiveness of the proposed theoretical results. Compared with the existing results, the main contributions of this paper are the following aspects: (i) The activation functions discussed in the fractional-order complex-valued delayed neural network model are no longer required to be differentiable. (ii) The impact of both leakage delays and discrete delays on uniform stability is taken into account for fractional-order complex-valued neural networks. (iii) Our results are less conservative and more general then the existed results.

Notations: The notations are quite standard. Throughout this paper, let $\mathbb{Z}^+$ denote the set of positive integers. Let i denote the imaginary unit, i.e. $i=\sqrt{-1}$. $\mathbb{C}^n$, $\mathbb{R}^{m\times n}$ and $\mathbb{C}^{m\times n}$ denote, respectively, the set of n-dimensional complex vectors, $m\times n$ real matrices and complex matrices. For complex matrix $W=(w_{jk})_{n\times n}\in\mathbb{C}^{n\times n}$, let $\|W\|=\sqrt{\sum_{j=1}^{n}\sum_{k=1}^{n}|w_{jk}|^2}$ be the norm of the matrix W. Let $C([-T,0],\mathbb{R}^n)$ denote the Banach space of continuous n-real vector functions defined on the interval $[-T,0]$, with the norm defined by $\|\varphi(t)\|=\max_{j=1,2,\ldots,n}\sup_{t\in[-T,0]}\{e^{-t}|\varphi_j(t)|\}$ for $\varphi\in C([-T,0],\mathbb{R}^n)$. For $x(t)=(x_1(t),x_2(t),\ldots,x_n(t))^T\in C([0,+\infty),\mathbb{R}^n)$, let $\|x(t)\|=\max_{j=1,2,\ldots,n}\sup_{t\in[0,+\infty)}\{e^{-t}|x_j(t)|\}$ be the norm of $x(t)$.

2. Problems formulation and preliminaries

In fractional calculus, the traditional definitions of the integral and derivative of a function are generalized from integer orders to arbitrary ones. In the time domain, the fractional-order derivative and integral operators are defined by a Laplace convolution operation as follows.

Definition 1 [47]**.** The Riemann–Liouville fractional integral of order $\alpha>0$ for a function $f:\mathbb{R}\to\mathbb{R}$ is defined as

$$D_{t_0}^{-\alpha}f(t)=\frac{1}{\Gamma(\alpha)}\int_{t_0}^{t}(t-\tau)^{\alpha-1}f(\tau)\mathrm{d}\tau,$$

where $\Gamma(\cdot)$ is the Gamma function, i.e. $\Gamma(\alpha)=\int_0^{\infty}t^{\alpha-1}e^{-t}\mathrm{d}t$.

Definition 2 [47]**.** The Riemann–Liouville fractional derivative of order $\alpha>0$ for a function $x:\mathbb{R}\to\mathbb{R}$ is defined as

$$^{RL}D_{t_0}^{\alpha}x(t)=\frac{1}{\Gamma(m-\alpha)}\frac{\mathrm{d}^m}{\mathrm{d}t^m}\int_{t_0}^{t}(t-\tau)^{m-\alpha-1}x(\tau)\mathrm{d}\tau,$$

where $m-1<\alpha<m$, $m\in\mathbb{Z}^+$, and $\Gamma(\cdot)$ is the Gamma function.

Definition 3 [47]**.** The Caputo fractional derivative of order $\alpha>0$ for a function $x:\mathbb{R}\to\mathbb{R}$ is defined as

$$^{C}D_{t_0}^{\alpha}x(t)=\frac{1}{\Gamma(m-\alpha)}\int_{t_0}^{t}(t-\tau)^{m-\alpha-1}x^{(m)}(\tau)\mathrm{d}\tau,$$

where $m-1<\alpha<m$, $m\in\mathbb{Z}^+$, and $\Gamma(\cdot)$ is the Gamma function.