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Numerical oscillation and non-oscillation for differential equation with piecewise continuous arguments of mixed type☆

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ABSTRACT

The paper deals with the oscillation and non-oscillation of the Runge–Kutta methods for a differential equation with piecewise continuous arguments of mixed type. The conditions of the oscillation and non-oscillation for the Runge–Kutta method are obtained. It is proved that oscillation of the analytic solution is not preserved by the Runge–Kutta method under any conditions. The conditions under which the non-oscillation of analytic solutions is preserved by the Runge–Kutta method are obtained. Some numerical experiments are given.

1. Introduction

Oscillatory behavior is one of the main considerations in the qualitative study of delay differential equations (DDEs) and is the subject of many investigations. In the last few decades the oscillatory theory of DDEs has been extensively developed. We refer to [1] and [2] for the general theory of oscillation. In recent years, there has been much interest in researching the differential equations with piecewise constant argument (EPCA) [3–12]. The strong interest in such equations is motivated by the fact that they represent a hybrid of continuous and discrete dynamical systems and combine the properties of both differential and difference equations. However, all of the researchers mostly pay attention to the oscillation of analytic solutions not numerical solutions, such as [13–21] and so on. As is known to all, various models in biology, mechanics, and electronics are developed by the numerical solutions of EPCA, such as [22]. Hence the study on the numerical solution of DDEs is needful. In 2007, professor Liu, Gao and Yang (see [23]) firstly investigated oscillation of numerical solution in the θ-methods for a kind of differential equations with piecewise constant argument, which is delay type, and the preservation of oscillation for θ-methods was studied. In 2009, Liu, Gao and Yang (see [24]) discussed the numerical oscillation of the same equation for Runge–Kutta methods, and also the preservation of oscillation and non-oscillation for Runge–Kutta methods are investigated. For a kind of nonlinear DDEs of population dynamics and a linear neutral DDEs, Gao and Liu (see [25,26]) investigated the numerical oscillations in 2011. In the same year, Wang et al. (see [27]) discussed stability and oscillation of EPCA of alternately advanced and retarded type and stability and oscillation of another type EPCA were studied by Song and Liu (see [28,29]) in 2012. In [30], Wang and Zhu investigated stability of EPCA of mixed type, the numerical oscillation is not discussed. In this paper, we will investigate the numerical oscillation for this kind of equations.

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The general form of EPCA is

$$\begin{aligned} x'(t) &= f(t, x(t), x(\alpha_1(t)), x(\alpha_2(t)), x(\alpha_3(t))), \quad t \geq 0, \\ x(-1) &= x_{-1}, x(0) = x_0, \end{aligned} \tag{1.1}$$

where the arguments $\alpha_i(t)$ $(i = 1, 2, 3)$ have intervals of constancy. In this paper, we consider the following differential equation with piecewise continuous argument (EPCA) of mixed type:

$$\begin{aligned} x'(t) &= px(t) + p_{-1}x([t-1]) + p_0x([t]) + p_1x([t+1]), \quad t \geq 0, \\ x(-1) &= x_{-1}, x(0) = x_0, \end{aligned} \tag{1.2}$$

where p, p_{-1}, p_0, p_1, x_{-1}, x_0 are real constants, $[\cdot]$ denotes the greatest integer function and $p_{-1} \neq 0$, $p_1 \neq 0$.

Definition 1.1 [31]**.** A solution of Eq. (1.2) on $[0, \infty)$ is a function $x(t)$ satisfying the conditions:

(1) $x(t)$ is continuous on $[0, \infty)$;
(2) The derivative $x'(t)$ exists at each point $t \in [0, \infty)$, with the possible exception of the points $[t] \in [0, \infty)$, where one-sided derivatives exist;
(3) Eq. (1.2) is satisfied on each interval $[n, n+1) \subset [0, \infty)$ with integral end-points.

The following theorem gives the solution of Eq. (1.2).

Theorem 1.2 [31]**.** *If $p \neq 0$, $p_{-1} \neq 0$ and $q_1 \neq 1$, then Eq. (1.2) has on $[0, \infty)$ a unique solution*

$$x(t) = m_{-1}(\{t\})c_{[t-1]} + m_0(\{t\})c_{[t]} + m_1(\{t\})c_{[t+1]}, \tag{1.3}$$

where $\{t\}$ is the fractional part of t and

$$c_{[t]} = \frac{\lambda_1^{[t+1]}(x_0 - \lambda_2 x_{-1}) + (\lambda_1 x_{-1} - x_0)\lambda_2^{[t+1]}}{\lambda_1 - \lambda_2}, \tag{1.4}$$

$$m_{-1}(t) = (e^{pt} - 1)p^{-1}p_{-1}, \quad m_0(t) = e^{pt} + (e^{pt} - 1)p^{-1}p_0, \quad m_1(t) = (e^{pt} - 1)p^{-1}p_1, \tag{1.5}$$

$$q_{-1} = m_{-1}(1), \quad q_0 = m_0(1), \quad q_1 = m_1(1), \tag{1.6}$$

λ_1 and λ_2 are the roots of equation

$$(1 - q_1)\lambda^2 - q_0\lambda - q_{-1} = 0.$$

Proof. Let $x_n(t)$ be the solution of Eq. (1.2) on the interval $[n, n+1)$. If we let $c_n = x(n)$ for integer n, then we have the equation

$$x_n'(t) = px_n(t) + p_{-1}c_{n-1} + p_0c_n + p_1c_{n+1} \tag{1.7}$$

with the solution

$$x_n(t) = e^{p(t-n)}c_n + p^{-1}(e^{p(t-n)} - 1)(p_{-1}c_{n-1} + p_0c_n + p_1c_{n+1}),$$

which can be written, by virtue of (1.5), as

$$x_n(t) = m_{-1}(t-n)c_{n-1} + m_0(t-n)c_n + m_1(t-n)c_{n+1}. \tag{1.8}$$

From (1.8) we can see that it suffices to know the constants c_n in order to determine $x(t)$. Taking into account

$$x_n(n+1) = x_{n+1}(n+1) = c_{n+1},$$

we obtain

$$c_{n+1} = m_{-1}(1)c_{n-1} + m_0(1)c_n + m_1(1)c_{n+1}, n \geq 0.$$

With the notations (1.6), this equation takes the form

$$(1 - q_1)c_{n+1} - q_0c_n - q_{-1}c_{n-1} = 0. \tag{1.9}$$

Its particular solution is sought as $c_n = \lambda^n$. Then

$$(1 - q_1)\lambda^2 - q_0\lambda - q_{-1} = 0. \tag{1.10}$$

Eq. (1.10) has two nontrivial solutions because of $p \neq 0$, $p_{-1} \neq 0$ and $q_1 \neq 1$. If the roots λ_1 and λ_2 of Eq. (1.10) are different, the general solution of Eq. (1.9) is

$$c_n = k_1\lambda_1^n + k_2\lambda_2^n,$$

with arbitrary constants k_1 and k_2. For $n = -1$ and $n = 0$, in view of $x(-1) = x_{-1}$ and $x(0) = x_0$, we have

$$k_1\lambda_1^{-1} + k_2\lambda_2^{-1} = x_{-1},$$