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A note on stability of hybrid stochastic differential equations

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ABSTRACT

In this paper, the *p*th moment stability of hybrid stochastic differential equations is investigated. Several new sufficient conditions are derived by constructing an auxiliary delayed differential equation and using the comparison principle. The proposed criteria remove some harsh restrictions imposed on the diffusion operators and improve some previous related works. Numerical examples and simulations are given to illustrate the effectiveness of theoretical results.

1. Introduction

Stochastic systems have come to play an important role in formulation and analysis in many branches of science and industry [1–6]. In these applications, the structure of stochastic system may experience abrupt changes caused by phenomena such as component failures, changing subsystem interconnections, and abrupt environmental disturbances [7]. For instance, appreciation and volatility rates in financial systems may be subject to random switches based on perceptions of investors and the air traffic management systems have to contend with aircraft mode switching together with some stochastic influences on aircraft dynamics, etc. Such systems have two components in their states. The first one evolves continuously in time, which can be represented by the classical stochastic differential equation (SDE). The second one takes values in a finite set and switches in a random manner between finite states. The hybrid stochastic differential equation (HSDE), which consists of both the logical switching rule driven by continuous-time Markov chains and the continuous state represented by SDEs, can provide an excellent mathematical modeling framework for these situations. Recently, HSDEs have been considered for the modeling of electric power systems, the control of a solar thermal central receiver, manufacturing processes and population dynamic [8,9], and the related theory of HSDEs has been developed very quickly [10–20].

As particular interest, the stability is always one of the most important issues in the theory of HSDEs [21]. Basak considered the stability of a semi-linear SDE with Markovian switching [22], while Mao studied the stability of nonlinear SDE with Markovian switching [23]. Both of them did not consider the time delay. However, in many real systems including manufacturing systems, telecommunication systems, population dynamics and network control systems, time delays often occur due to the limitation of transmission or switching speeds. It is well known that the delay may cause oscillation and instability in systems. Therefore, it is of great significance to study the stability of HSDEs with time delays, and lots of related literature has been published [24–37]. For example, by using nonnegative semi-martingale convergence theorem, some almost sure stability criteria have been derived for HSDEs with delays [24–26]. In [27–31], the stability in mean square of HSDEs with delays have been studied by constructing Lyapunov–Krasovskii functionals. In [32–37], a few of interesting results on the moment asymptotic stability and moment exponential stability of HSDEs have been derived by using Lyapunov functions and Razumikhin techniques.

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It is worth noting that the stability analysis of HSDEs with delays has not been fully investigated. First, those results in [24–26] required the time delay to be a constant or a differentiable function with the derivative of which is bounded by a constant number. In addition, some of existing results required that the diffusion operator of the Lyapunov function along the system $\mathcal{L}V$ is negative definite (see [32–35]) or bounded by the linear form $-p(t)V+q(t)V(t-\tau)$ with $p(t)-q(t)\geq\sigma>0$ for all t (see [36]). However, time delays may occur in an irregular fashion, and sometimes they may be time-varying and not continuously differentiable such as $|\cos t|$. Furthermore, we know that it is always a very difficult task to find a suitable Lyapunov function for those nonlinear HSDEs with time-varying coefficients. When the diffusion operator $\mathcal{L}V$ is allowed to be positive in some intervals or time-varying coefficients of the upper-bound with linear form are no longer satisfied $p(t)-q(t)\geq\sigma>0$ all the time, for instance $p(t)=\sin t$ and $q(t)=0$, those existing works in [24–36] can not be applied. In this case, how can we study the stability of HSDEs with delays? To the best of the author's knowledge, very few result has been reported to deal with such situations, which are still very challenging and remain as open issues.

Motivated by the above discussion, we see the necessity to develop some new stability criteria for HSDEs with delays. In this paper, by means of the auxiliary delayed differential equation and the comparison principle, we establish several sufficient conditions to check the stability, asymptotic stability and exponential stability in pth moment for the trivial solution of HSDEs with delays. Our results show that the diffusion operator does not need to be negative definite all the time and the corresponding upper-bound may take a much more general form, which make the proposed method be more applicable. At last, two numerical examples are provided to illustrate the effectiveness of theoretical results.

2. Preliminaries

Let $\mathbb{R}=(-\infty,+\infty)$, $\mathbb{R}_a^+=[a,+\infty)$ and $\mathbb{N}=\{1,2,\cdots\}$. $\mathbb{R}^n$ denotes the space of n-dimensional real column vectors and $\mathbb{R}^{m\times n}$ represents the class of $m\times n$ matrices with real components. $|\cdot|$ denotes the Euclidean vector norm in $\mathbb{R}^n$. Let $\mathcal{K}$ denote the class of continuous strictly increasing functions μ from $\mathbb{R}_0^+$ to $\mathbb{R}_0^+$ with $\mu(0)=0$ and $\mathcal{K}_\infty$ denote the class of functions $\mu\in\mathcal{K}$ with $\mu(s)\to+\infty$ as $s\to+\infty$. We also denote by $\mu\in\mathcal{VK}$ and $\mu\in\mathcal{CK}$ if $\mu\in\mathcal{K}$ is convex and concave, respectively. Let Ψ_a be the family of all bounded continuous functions ψ from $\mathbb{R}$ to $\mathbb{R}$ satisfying $\psi(s)\geq a$ for some real number a. For a function $\psi(t)$, we denote by $[\psi(t)]^+=\max\{\psi(t),0\}$ and $[\psi(t)]^-=\min\{\psi(t),0\}$. For some $\tau>0$, let $\mathcal{C}=\mathcal{C}([-\tau,0];\mathbb{R}^n)$ be the family of all continuous $\mathbb{R}^n$-valued functions ϕ defined on $[-\tau,0]$ with norm $||\phi||=\sup_{s\in[-\tau,0]}|\phi(s)|$.

Let $(\Omega,\mathcal{F},\{\mathcal{F}_t\}_{t\geq0},\mathbf{P})$ be a complete probability space with a natural filtration $\{\mathcal{F}_t\}_{t\geq0}$ satisfying the usual conditions (i.e., it is right continuous and $\mathcal{F}_0$ contains all $\mathbf{P}$-null sets). We denote by $\mathcal{L}^p_{\mathcal{F}_t}([-\tau,0];\mathbb{R}^n)$ the family of all $\mathcal{F}_t$-measurable $\mathcal{C}$-valued stochastic processes $\xi=\{\xi(\theta):\theta\in[-\tau,0]\}$ satisfying $||\xi||^p_{\mathcal{L}^p}=\sup_{\theta\in[-\tau,0]}\mathbf{E}|\xi(\theta)|^p<\infty$, where $\mathbf{E}$ denotes the expectation operator. Let $w(t)=(w_1(t),\dots,w_n(t))^T$ be an n-dimensional Brownian motion defined on the probability space $(\Omega,\mathcal{F},\{\mathcal{F}_t\}_{t\geq0},\mathbf{P})$. Let $\{r(t),t\in\mathbb{R}_0^+\}$ be a right continuous Markov chain on the same probability space taking values in a finite state space $\mathcal{M}=\{1,2,\dots,m\}$ with generator $\Gamma=(\gamma_{ij})_{m\times m}$ given by

$$\mathbf{P}(r(t+\delta)=j|r(t)=i)=\begin{cases}\gamma_{ij}\delta+o(\delta), & i\neq j,\\ 1+\gamma_{ij}\delta+o(\delta), & i=j,\end{cases}$$

in which $\delta>0$ and $\lim_{\delta\to0^+}\frac{o(\delta)}{\delta}=0$. Here $\gamma_{ij}\geq0$ is the transition rate from i to j if $i\neq j$, while $\gamma_{ii}=-\sum_{i\neq j}\gamma_{ij}$. We always assume $r(t)$ is independent of $w(t)$. It is known that almost all sample paths of $r(t)$ are right-continuous step functions with a finite number of simple jumps in any finite subinterval of $\mathbb{R}_0^+$.

Consider the following HSDE

$$dx(t)=f(t,r(t),x_t)dt+\sigma(t,r(t),x_t)dw(t),\quad t\geq t_0, \tag{2.1}$$

and the initial condition

$$x_{t_0}=\phi\in\mathcal{L}^p_{\mathcal{F}_{t_0}}([-\tau,0];\mathbb{R}^n),\quad r(t_0)=i_0\in\mathcal{M}, \tag{2.2}$$

where x_t is regarded as $\mathcal{C}$-valued stochastic process $x_t(\theta)=x(t+\theta)$ for $\theta\in[-\tau,0]$, $f:\mathbb{R}^+_{t_0}\times\mathcal{M}\times\mathcal{C}\to\mathbb{R}^n$, $\sigma:\mathbb{R}^+_{t_0}\times\mathcal{M}\times\mathcal{C}\to\mathbb{R}^{n\times n}$ are measurable functions and satisfy necessary conditions so that, for any initial condition (2.2), Eq. (2.1) has a unique global solution $x(t)=x(t,t_0,\phi)$, $t\geq t_0$. Moreover, we assume that $f(t,i,0)=0$ and $\sigma(t,i,0)=0$ for all $t\in\mathbb{R}^+_{t_0}$, $i\in\mathcal{M}$, which implies Eq. (2.1) admits a trivial solution $x(t)=0$.

Remark 2.1. Let $r(t)=i\in\mathcal{M}$. Eq. (2.1) can be regarded as the result of the following m SDEs

$$dx(t)=f(t,i,x_t)dt+\sigma(t,i,x_t)dw(t),\quad t\geq t_0,$$

switching from one mode to the others according to the movement of the Markov chain $r(t)$.

Remark 2.2. The $\sigma(t,r(t),x_t)$ in Eq. (2.1) is called to be the diffusion coefficient, which is used to represent the intensity of white noise suffered by the system. In general, $\sigma(t,r(t),x_t)$ has it's own physics meaning in practical problems. For example, the $\sigma(t,r(t),x_t)$ represents the force exerted on the particle by the molecular collisions in the Brownian motion of a particle in a fluid while for a stochastic Lotka–Volterra ecosystem in a random environment, it represents the noise intensities in different external environments. The function $x(t)$ satisfying the Eq. (2.1) is a continuous Markov process. For more detailed information about $x(t)$, we refer the reader to Definition 3.11 on page 88 in [10].