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Controllability of measure driven evolution systems with nonlocal conditions

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ABSTRACT

This paper investigates complete controllability of semilinear measure driven differential systems with nonlocal conditions. Without assuming the compactness of the evolution system related to the linear part of the measure system, some sufficient conditions for controllability are established by using the measure of noncompactness and the Mönch fixed point theorem. The results obtained here improve and generalize many known results.

1. Introduction

Measure differential equations or measure driven equations are applied to model the dynamical systems with discontinuous trajectories. The critical distinction between this type of equations and usual impulsive differential equations is that the former permit an infinite number of discontinuous points in a finite time interval and hence can model some non-classical phenomena such as the quantum effects and Zeno trajectories [1,2]. One can refer to the literatures [3–7] for finite-dimensional-state-valued measure differential equations and [8–10] for infinite-dimensional-state-valued measure differential equations.

On the other hand, complete controllability of several kinds of nonlinear dynamical systems such as impulsive differential equations, fractional order dynamic systems, stochastic systems has been investigated extensively (see [11–23] and the references therein). However, to the best of our knowledge, there have not been any results concerning to complete controllability of measure driven equations in Banach spaces. This paper will fill this gap and discuss complete controllability of semilinear measure driven evolution systems with nonlocal conditions. If the evolution operator related to the linear part of the system is compact, then the control operator can not be a surjection [24]. Therefore, it is necessary to relax the restriction on the evolution operator. In this paper, we will provide the complete controllability criteria of the discussed system by using the Hausdorff measure of noncompactness and the Mönch fixed point theorem. Because solutions of measure driven equations are regulated functions, we have to discuss the problem in the space of regulated functions.

Consider the following measure driven equation of the form

$$\begin{cases} dx(t) = A(t)x(t) + Bu(t) + f(t, x(t))dg(t), & t \in J; \\ x(0) + p(x) = x_0. \end{cases} \tag{1}$$

where $J = [0, b]$ with $b > 0$. The state variable $x(\cdot)$ takes values in Banach space X with the norm $\|\cdot\|$. $A(t)$ is a family of linear operators which generates an evolution system $\{\mathcal{U}(t, s): 0 \le s \le t \le b\}$. f: $J \times X \to X$ and p: $G(J; X) \to X$ will be

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specified later, where $G(J; X)$ denotes the space of regulated functions on J in which we consider the problem. The control function $u(\cdot)$ is given in $L^2(J; V)$, a Banach space of admissible control functions with V as a Banach space. B is a bounded linear operator mapping V into X. $g: J \to \mathbb{R}$, is a nondecreasing function continuous from the left. dx and dg denote the distributional derivatives of the state variables and the function g, respectively.

This paper is organized as follows. In Section 2, we recall some concepts and results about regulated functions and the Hausdorff measure of noncompactness together with a two-parameter family of bounded linear operators. Main results are provided in Section 3 and an example is also given to illustrate our results. Finally, some conclusions are drawn in Section 4.

2. Preliminaries

We first recall some concepts and basic results which will be used throughout this paper.

Let X be a Banach space with the norm $\|\cdot\|$ and $J = [0, b]$ a closed interval of the real line. A function $f: J \to X$ is called regulated on J, if the limits

$$\lim_{s\to t^-} f(s) = f(t^-), t \in (0, b] \quad \text{and} \quad \lim_{s\to t^+} f(s) = f(t^+), t \in [0, b)$$

exist and are finite (see [25,26]). The space composed of all regulated functions $f: J \to X$ is denoted by $G(J; X)$. It is well known that the set of discontinuities of a regulated function is at most countable and that the space $G(J; X)$ is a Banach space endowed with the norm $\|f\|_\infty = \sup_{t\in J} \|f(t)\|$ (see [25]).

Lemma 2.1. *[27] Consider the functions $f: J \to X$ and $g: J \to \mathbb{R}$ such that g is regulated and $\int_0^b f dg$ exists. Then for every $t_0 \in [0, b]$, the function $h(t) = \int_{t_0}^t f dg$, $t \in [0, b]$, is regulated and satisfies*

$$h(t^+) = h(t) + f(t)\Delta^+ g(t), \quad t \in [0, b),$$
$$h(t^-) = h(t) - f(t)\Delta^- g(t), \quad t \in (0, b],$$

where $\Delta^+ g(t) = g(t^+) - g(t)$ and $\Delta^- g(t) = g(t) - g(t^-)$. $g(t^-)$ and $g(t^+)$ denote the left limit and the right limit of the function g at the point t, respectively.

Definition 2.2. [26] A set $\mathcal{A} \subset G(J; X)$ is called equiregulated, if for every $\varepsilon > 0$ and $t_0 \in J$, there is a $\delta > 0$ such that

(i) If $x \in \mathcal{A}$, $t \in J$ and $t_0 - \delta < t < t_0$, then $\|x(t_0{}^-) - x(t)\| < \varepsilon$.
(ii) If $x \in \mathcal{A}$, $t \in J$ and $t_0 < t < t_0 + \delta$, then $\|x(t) - x(t_0{}^+)\| < \varepsilon$.

Lemma 2.3. *[26] Let $\{x_n\}_{n=1}^\infty$ be a sequence of functions from J to X. If x_n converges pointwisely to x_0 as $n \to \infty$ and the sequence $\{x_n\}_{n=1}^\infty$ is equiregulated, then x_n converges uniformly to x_0.*

Lemma 2.4. *[10] Let $W \subset G(J; X)$. If W is bounded and equiregulated, then the set $\overline{co}(W)$ is also bounded and equiregulated.*

Let X be a Banach space. The Hausdorff measure of noncompactness of a bounded subset S of X is defined to be the infimum of the set of all real numbers $\varepsilon > 0$ such that S can be covered by a finite number of balls of radius smaller than ε, that is,

$$\beta(S) = \inf\{\varepsilon > 0 : S \subset \cup_{i=1}^n B(\xi_i, r_i), \xi_i \in X, r_i < \varepsilon (i = 1, \dots, n), n \in \mathbb{N}\}.$$

where $B(\xi_i, r_i)$ denotes the open ball centered at ξ_i and of radius r_i.

Lemma 2.5. *[28,29] Let S, T be bounded subsets of X and $\lambda \in \mathbb{R}$. Then*

(i) *$\beta(S) = 0$ if and only if S is relatively compact;*
(ii) *$S \subseteq T$ implies $\beta(S) \le \beta(T)$;*
(iii) *$\beta(\overline{S}) = \beta(S)$;*
(iv) *$\beta(S \cup T) = \max\{\beta(S), \beta(T)\}$;*
(v) *$\beta(\lambda S) = |\lambda|\beta(S)$, where $\lambda S = \{x = \lambda z : z \in S\}$;*
(vi) *$\beta(S + T) \le \beta(S) + \beta(T)$, where $S + T = \{x = y + z : y \in S, z \in T\}$;*
(vii) *$\beta(co(S)) = \beta(S)$;*
(viii) *If the map $Q: D(Q) \subseteq X \to Z$ is Lipschitz continuous with a constant k, then $\beta_Z(Q\Omega) \le k\beta(\Omega)$ for any bounded subset $\Omega \subseteq D(\Omega)$, where Z is a Banach space.*

Let W be a subset of $G(J; X)$. For each fixed $t \in J$, we denote $W(t) = \{x(t) : x \in W\}$. Next we will present some results of the Hausdorff measure of noncompactness in the space of regulated functions $G(J; X)$.

Lemma 2.6. *[10] Let $W \subset G(J; X)$ be bounded and equiregulated on J. Then $\beta(W(t))$ is regulated on J.*

Lemma 2.7. *[10] Let $W \subset G(J; X)$ be bounded and equiregulated on J. Then*

$$\beta(W) = \sup\{\beta(W(t)) : t \in J\}.$$

Denote by $\mathcal{LS}_g(J; X)$ the space of all functions $f: J \to X$ that are Lebesgue–Stieltjes integrable with respect to g. Let μ_g be the Lebesgue–Stieltjes measure on J induced by g. Since the Lebesgue–Stieltjes measure is a regular Borel measure, then the following result holds by Theorem 3.1 in [30].