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Forts of quadratic polynomials under iteration

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Abstract

Since some dynamical behaviors of a one-dimensional mapping are influenced by the number of forts, attention is paid to the change of the number under iteration. For simple computation we work on quadratic polynomials. We use the theory of polynomial complete discrimination system to give a symbolic algorithm for the number of forts of iterated polynomials and apply the algorithm to quadratic functions, which proves an alternative result that the number either persists to be 1 or tends to infinity under iteration. We further compute the number for iterates of order $2, 3, ..., 7$ in the above infinity case and obtain critical values of the parameter at which the number changes. Those changes with finitely many data display a conjectured Fibonacci rule.

Keywords: iteration; forts; quadratic polynomial; complete discrimination system; Fibonacci sequence

MSC (2010) Classification: 37E05; 39B12; 68W30.

1 Introduction

Consider a mapping $f : E \to E$, where E is a nonempty set. For a given integer $n \geq 1$, the n-th order iterate f^n of f is defined by

$$f^n(x) = f(f^{n-1}(x)), \quad f^0(x) = x, \quad \forall x \in E,$$

recursively. Iteration is one of the most important actions in engineering techniques because of the extensive applications of numerical computation

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