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Numerical solution of nonlinear singular boundary value problems

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Abstract: In this paper, an efficient method based on the simplified reproducing kernel method(SRKM) and least squares method(LSM) is proposed for solving nonlinear singular boundary value problems. The algorithm consists of two steps. First, we apply SRKM to solve a linear equation which contains a set of parameters, second the LSM is used to find the optimal parameters. Error estimation and convergence order for the presented method are also discussed. Our numerical experiments validate our theoretical findings and demonstrate the performance of the algorithm in terms of simplicity, accuracy, and efficiency.

Keywords: Nonlinear singular boundary value problem, simplified reproducing kernel method, least squares method, error analysis, convergence order.

1 Introduction

In the present work, the nonlinear singular boundary value problems will be considered

$$\begin{cases} u''(x) + \frac{\alpha}{x}u'(x) = f(x, u(x)), \quad 0 < x \leq 1, \\ u'(0) = 0, au(1) + bu'(1) = c, \end{cases} \tag{1.1}$$

where $\alpha \geqslant 1$, $a > 0$, $b \geqslant 0$ and c are any finite real constant. Nonlinear singular boundary value problems arise in many areas of physiology, such as: chemical reaction, gas dynamics, electro hydrodynamics, nuclear physics, atomic structures and atomic calculations[1-5]. Of special interest is the case when $\alpha = 2$ and $f(x, u) = \frac{nu}{u+k}, n > 0, k > 0$ in Eq.(1.1) which arises in the modeling of steady state oxygen diffusion in a spherical cell with Michaelis-Menten uptake kinetics [1]. Another case is when $\alpha = 2$ and $f(x, u) = -ke^{-nu}, k > 0, n > 0$ which occurs in the formulation of the distribution of heat sources in the human head [2]. Eq.(1.1) with $\alpha = 2$ and $f(x, u) = \gamma e^{u}$, where γ is a physical parameter that arises in the theory of electro-hydrodynamics [3]. For $\alpha = 1$ and $f(x, u) = u^{\gamma}$, where γ is a physical constant, the above equation is used to study thermal explosions [4].

In reference [6,7] the authors proved existence and uniqueness of solutions for the problem Eq.(1.1). Since Eq.(1.1) has singularity at $x = 0$ that is the main difficulty. Various numerical methods have been used to solve this singular boundary value problem, for example: cubic spline and finite difference

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